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Analyze the moment of inertia of an object in uniformly accelerated linear motion and angular motion

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Abstract. In studying particle kinematics, calculus plays an important role in analyzing and solving motion problems as a key mathematical tool. This paper mainly discusses the application of calculus in uniformly accelerated linear motion (LM) and angular motion (AM), focusing on the relationship between position, velocity, and acceleration and the analysis of the rate of change of motion. Through the study of uniformly accelerated LM, this paper guides researchers to use calculus to establish the mathematical relationship between position, velocity, and acceleration and verifies the accuracy of these formulas through practical cases. Similarly, in AM, this paper provides an in-depth understanding of the dynamic behavior of an object rotating around a fixed axis through calculus and calculates and applies important parameters in AM to solve practical problems by analyzing angular velocity and acceleration. In addition, this paper demonstrates the potential of calculus for a wide range of applications in dealing with variable speed motion and rate of change analysis. By integrating the rate of change of displacement and acceleration, it provides a precise tool for the optimization and resolution of engineering problems. In conclusion, calculus not only enriches the theory of kinematics but also demonstrates its important value in practical applications.

Keywords: Calculus, particle kinematics, uniformly accelerated linear motion, angular motion.

1. Introduction

In physics, the motion of objects is a basic and important research field, especially uniform linear motion (LM) and angular motion (AM). These two forms of motion have a wide range of significance in practical applications, such as the acceleration and deceleration of vehicles, the rotation of mechanical parts, etc. Understanding the basic laws and characteristics of these motions not only helps to explain and predict the motion behavior of objects but also provides theoretical support for engineering design and scientific research. Moment of inertia, as an important physical quantity that describes the rotational characteristics of objects, is crucial to the analysis and understanding of AM. In the analysis of uniform LM and AM, calculus tools can provide precise mathematical descriptions and solutions, allowing us to deeply understand the behavior of objects in these motion states.

Uniformly accelerated LM is a basic concept in classical mechanics, and its historical evolution can be traced back to ancient Greece. Early scientists, such as Aristotle, had a preliminary understanding of motion phenomena, but due to the lack of systematic mathematical tools, their theories mostly remained at the level of intuitive description. With the development of mathematics and physics, especially the

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invention of calculus, the theoretical basis of uniformly accelerated LM has gradually been improved [1, 2].

In short, the historical evolution of uniformly accelerated LM shows the process of scientific theory from preliminary description to system analysis. This process not only reflects the development of physics but also reflects the core role of mathematical tools in scientific research. By understanding these historical backgrounds, we can better grasp the modern application and research direction of uniformly accelerated LM.

The concept of angular momentum plays an important role in physics, and its development reflects the evolution of classical mechanics and modern physics. Angular momentum describes the state of motion of an object rotating around a certain axis and is one of the core quantities for studying rotational motion and torque effects. Its history can be traced back to ancient times, but the truly systematic understanding and application are the result of centuries of efforts by scientists.

The earliest form of angular momentum can be traced back to ancient Greek astronomers and physicists such as Aristotle and Ptolemy, who indirectly involved the concept of angular momentum when describing the motion of celestial bodies. However, these ancient descriptions were mostly qualitative and lacked mathematical rigor [3, 4].

The concept of angular momentum is deeply rooted in the history of science, tracing its origins back to the intuitive notions of rotation and circular motion observed by ancient civilizations. Over the centuries, this concept has been refined and formalized, evolving into a fundamental principle in both classical and modern physics.

The journey of angular momentum began with the rudimentary observations of celestial bodies by ancient astronomers. They noted the circular paths of stars and planets, which laid the groundwork for early theories of motion and force. The formal study of angular momentum, however, started with the work of scientists like Galileo and Kepler, who described the motions of planets and other celestial objects. It was Isaac Newton who later solidified the concept by incorporating it into his laws of motion and universal gravitation, explaining how angular momentum is conserved in a closed system.

In the 19th century, the development of classical mechanics further expanded our understanding of angular momentum. Scientists like Leonhard Euler and Joseph-Louis Lagrange introduced mathematical frameworks that treated rotational motion in terms of angular velocity and moment of inertia, paving the way for more complex applications in engineering and technology.

The advent of quantum mechanics in the early 20th century marked a significant leap in the application of angular momentum. The theory introduced the idea of quantized angular momentum, which has discrete values, a concept crucial for understanding atomic and subatomic particles. This quantization explained the spectral lines of atoms and led to the development of the electron shell model by Niels Bohr and others, fundamentally altering our comprehension of chemical bonding and atomic structure.

Today, the principles of angular momentum are integral to numerous fields, including astronomy, where they are used to study the dynamics of galaxies and black holes; in engineering, particularly in the design of gyroscopes and other devices that stabilize and navigate aircraft and spacecraft; and in emerging technologies like quantum computing, where understanding the angular momentum of particles is essential for developing new types of information processing systems.

Understanding the historical development of angular momentum not only highlights its importance in the advancement of various scientific disciplines but also underscores its ongoing relevance in tackling modern scientific and technological challenges. This historical perspective enriches our appreciation of angular momentum's role in the broader narrative of physics, where past insights continually inform future discoveries [5].

This study aims to analyze the moment of inertia of an object in uniformly accelerated LM and AM by combining calculus knowledge. This analysis is of great significance for a deep understanding of the motion characteristics of an object and for improving the technical level in related fields. First, the calculus method can accurately calculate the motion trajectory and velocity change of an object under different accelerations, which is particularly important for applications in mechanical engineering,

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transportation and other fields. Secondly, in AM, the analysis of the moment of inertia can reveal how an object distributes and transfers its momentum during rotation, thereby affecting rotational stability and dynamic characteristics. Further, this analysis can help design more efficient and stable mechanical systems, such as flywheels, rotating equipment, etc. In addition, this study also provides more in-depth mathematical tools for physics education, enabling students to better understand the complexity and diversity of motion. By combining calculus and the analysis of the moment of inertia, this study not only expands the scope of the application of theoretical knowledge but also provides strong support for the solution of practical engineering problems.

2. Theoretical basis

2.1. Basic concepts

2.1.1. Definition

Recti LM: characterized by an object's trajectory along a straight line, represents one of the most fundamental concepts in classical mechanics. This type of motion is foundational in understanding how forces affect motion in a simplified, one-dimensional space. By studying recti LM, physicists can develop models that predict the behavior of objects under various force conditions, serving as a stepping stone to more complex multi-dimensional motion analyses. Variable LM, on the other hand, involves changes in the speed of an object as it moves along a straight path. This motion is more complex because it implies the presence of net forces acting on the object, causing it to accelerate or decelerate. The study of variable LM is crucial for understanding how objects behave under the influence of varying forces. The equations of motion derived from Newton's laws provide powerful tools for analyzing such movements. Understanding these different types of recti LM is essential not only for physics but also for various practical applications across engineering, astronomy, and even economics, where linear models can often describe trends and changes within systems. The principles learned from recti LM apply broadly, aiding in the comprehension of more intricate systems involving curvilinear and multidimensional motions. Thus, the study of an object's movement along a straight line serves as a fundamental component of the broader discipline of mechanics, illuminating the universal principles that govern motion in the physical world.

AM: an integral aspect of rotational dynamics, plays a crucial role in the understanding of systems where objects move circularly or rotationally around a fixed axis. This type of motion is fundamental in numerous scientific and engineering applications, from the design of mechanical systems like engines and turbines to the study of celestial bodies and their orbits. Angular Velocity is a measure of the rate at which an object rotates around an axis. It is often expressed in radians per second and provides a clear picture of how quickly an object is spinning. Angular velocity is vectorial, Angular acceleration is particularly important when analyzing scenarios where rotational speeds vary, such as a spinning top slowing down due to friction, or an electric motor picking up speed as it powers up Different shapes and mass distributions around the axis will have different moments of inertia, influencing how they rotate under similar forces. These concepts are not just academic; they have practical implications in everyday technology and natural phenomena. For example, engineers use knowledge of AM to design wheels and gears that operate efficiently within mechanical systems. In sports, understanding AM helps athletes optimize their movements in sports like figure skating and discus throw, where rotation is a key component of performance. Additionally, astronomers apply these principles to study the rotational dynamics of planets, stars, and galaxies to better understand their evolution and structure. Furthermore, AM is critical in the study of systems that balance rotational and translational movements, such as in robotic arms or the flight dynamics of spacecraft and aviation. These systems often require precise calculations of angular velocities, accelerations, and moments of inertia to ensure stability and control during operation. Overall, AM forms a foundational concept in the study of dynamics, offering insights into how objects rotate and behave under various forces, enhancing our ability to design, predict, and control physical systems in both terrestrial and astronomical contexts.

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2.1.2. Relative formulas for LM

Formula for uniform LM: $s(t) = s_0 + vt$. $s(t)$ is time t the position of the object at s_0 is the initial position, v is a constant speed.

The formula for uniformly accelerated LM is:

$$v(t) = v_0 + at \tag{1}$$

$$s(t) = s_0 + v_0 t + \frac{1}{2}at^2 \tag{2}$$

$$v^2 = v_0^2 + 2a(s - s_0) \tag{3}$$

Where, $v(t)$ is time t The speed of v_0 is the initial velocity, a is a constant acceleration, $s(t)$ is time t The position ats_0 is the initial position.

AM related formula:

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \tag{4}$$

$$\omega(t) = \omega_0 + \alpha t \tag{5}$$

$$\tau = I\alpha \tag{6}$$

Where, $\theta(t)$ is time t the angular displacement at $\omega(t)$ is the angular velocity, α is the angular acceleration, τ is the torque, I is the moment of inertia.

2.2. Knowledge analysis combined with calculus

2.2.1. Uniformly accelerated LM [6]

Calculus plays an important role in analyzing uniformly accelerated LM. a From the initial speed v_0 Start moving, and the change of its position over time can be solved by integration.

Integral of velocity: In uniformly accelerated LM, acceleration a is constant, so the relationship between speed and time can be expressed as:

$$v(t) = v_0 + at \tag{7}$$

This can be found by integrating the acceleration:

$$v(t) = \int a\,dt + v_0 \tag{8}$$

Integration of position: The change of the position of an object over time can be found by integrating the velocity:

$$s(t) = s_0 + \int v(t)\,dt \tag{9}$$

The speed formula $v(t) = v_0 + at$ Substituting, it get:

$$s(t) = s_0 + \int (v_0 + at)\,dt = s_0 + v_0 t + \frac{1}{2}at^2 \tag{10}$$

This formula describes the time course of an object. t The position at that time.

Speed-position relationship: Using calculus, the relationship between speed and position can be expressed as:

$$v^2 = v_0^2 + 2a(s - s_0) \tag{11}$$

This is obtained by substituting the velocity formula into the position integral formula.

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2.2.2. Moment of inertia in AM

Moment of Inertia I is an important physical quantity that describes the inertia of an object rotating around a certain axis. In AM, the moment of inertia can be calculated by integration, which represents the influence of how the mass of an object is distributed relative to the axis of rotation [7].

Calculation of moment of inertia: For an object with a discrete mass distribution, the moment of inertia is defined as:

$$I = \sum m_i r_i^2 \tag{12}$$

Where, m_i The first i is the mass of a small particle, r_i is its distance to the axis of rotation.

Continuous Mass Distribution: For an object with a continuous mass distribution, the moment of inertia can be calculated by integrating the mass elements:

$$I = \int r^2 \, dm \tag{13}$$

Where, r is the distance of the mass element from the axis of rotation, dm is the mass element. This integral formula is used to calculate the moment of inertia of objects with complex shapes.

Application example: For example, for a thin disk rotating around its central axis, its moment of inertia is:

$$I = \frac{1}{2} m r^2 \tag{14}$$

Where, m is the mass of the disk, r is the radius of the disk. This formula is obtained by integrating the mass elements on the disk and reflects the relationship between the moment of inertia and the shape and mass distribution of the object.

Combining the knowledge of calculus to analyze the moment of inertia in uniformly accelerated LM and AM can provide a more accurate and comprehensive understanding, helping us to apply these theories to perform more complex calculations and designs in practical engineering and scientific research [8, 9].

2.3. Application cases

2.4. Analysis of several variable speed movements

The analysis of variable speed motion is an important part of understanding the motion behavior of an object and optimizing its design. The following is an analysis of several common variable speed motions.

- Uniformly accelerated LM

Assume that the initial velocity of the object is v_0, the acceleration is a, at time t the velocity and position of an object can be calculated using the following formula:

$$v(t) = v_0 + at \tag{15}$$

$$s(t) = s_0 + v_0 t + \frac{1}{2} a t^2 \tag{16}$$

Case analysis: Consider a car accelerating from a stationary state, assuming the acceleration is 3 m/s^2, go through 10 After 1 second, the speed of the car is:

$$v(10) = 0 + 3 \times 10 = 30 \tag{17}$$

The car is here 10 The distance traveled in seconds is:

$$s(10) = 0 + 0 \times 10 + \frac{1}{2} \times 3 \times 10^2 = 150 \tag{18}$$

- Uniform deceleration LM

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Assume that the initial velocity is v_0, the deceleration is $-a$, at time t The velocity and position of an object can be calculated using the following formula:

$$v(t) = v_0 - at \tag{19}$$

$$s(t) = s_0 + v_0 t - \frac{1}{2}at^2 \tag{20}$$

Case Study: A car with an initial velocity 20 m/s Start to decelerate, the deceleration rate is 2 m/s^2,go through 5 After 1 second, the speed of the car is:

$$v(5) = 20 - 2 \times 5 = 10 \tag{21}$$

The car is here 5 The distance traveled in seconds is:

$$s(5) = 0 + 20 \times 5 - \frac{1}{2} \times 2 \times 5^2 = 75 \tag{22}$$

- Angular acceleration

Assume that the initial angular velocity is ω_0, the angular acceleration is α, at time t The angular velocity and angular displacement of an object can be calculated by the following formula:

$$\omega(t) = \omega_0 + \alpha t \tag{23}$$

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \tag{24}$$

Case analysis: Suppose a wheel starts to accelerate from rest, and the angular acceleration is 4 rad/s^2,go through 3 After seconds, the angular velocity of the wheel is:

$$\omega(3) = 0 + 4 \times 3 = 12 \tag{25}$$

Wheels in 3 The angular displacement per second is:

$$\theta(3) = 0 + 0 \times 3 + \frac{1}{2} \times 4 \times 3^2 = 18 \tag{26}$$

These formulas and calculation methods can be used to analyze various practical problems, such as evaluating vehicle acceleration performance, predicting the trajectory of moving objects, etc. In engineering applications, traffic design, and other scientific research fields, the theory and calculation methods of uniform LM provide important tools and foundations. The concept of uniform LM is widely used in fields such as traffic engineering and mechanical design. For example, when designing traffic lights, engineers use the uniform LM model to calculate the passing time of vehicles when the lights change. In addition, the design of conveyor belts in the manufacturing industry also uses the theory of uniform LM to ensure the smooth transmission of objects on the production line.

2.5. Application of calculus ideas in high school physics dynamics

Calculus ideas are widely used in high school physics dynamics, especially in describing the rate of change of motion, calculating displacement, and solving dynamic problems. Here are some key applications:

- Relationship between position, velocity, and acceleration

Calculus is used to describe the relationships between states of motion of an object. The velocity of an object is the derivative of position concerning time, and acceleration is the derivative of velocity concerning time. Using calculus, these relationships can be expressed as formulas:

$$v(t) = \frac{d}{dt}s(t) \tag{27}$$

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$$a(t) = \frac{d}{dt}v(t) = \frac{d^2}{dt^2}s(t) \tag{28}$$

These formulas help students understand how to derive velocity and acceleration functions from position functions.

- Calculation of displacement

In uniformly accelerated LM, the displacement of an object can be obtained by integrating the velocity function. For example, in uniformly accelerated LM, the velocity function is:

$$v(t) = v_0 + at \tag{29}$$

By integrating the velocity function, the displacement function can be obtained:

$$s(t) = s_0 + \int (v_0 + at)\, dt = s_0 + v_0 t + \frac{1}{2}at^2 \tag{30}$$

This helps students understand how to derive displacement from velocity and solve real-world problems.

- Calculation of Kinetic Energy and Work

Calculus ideas are also applied to the calculation of kinetic energy and work. For example, work is the integral of force over displacement:

$$W = \int F\, ds \tag{31}$$

The kinetic energy theorem also involves integral calculations in mechanics:

$$\Delta K = \int F\, dx \tag{32}$$

Through these formulas, students can understand how to calculate the change in energy of an object under the action of a force through integration.

2.5.1. Applications of AM

In AM, the relationship between angular displacement, angular velocity, and angular acceleration can also be expressed using calculus. For example, angular velocity is the time derivative of angular displacement:

$$\omega(t) = \frac{d}{dt}\theta(t) \tag{33}$$

Angular acceleration is the time derivative of angular velocity:

$$\alpha(t) = \frac{d}{dt}\omega(t) = \frac{d^2}{dt^2}\theta(t) \tag{34}$$

These formulas are used to calculate various physical quantities in AM and help students solve practical problems.

Through these application cases and the practical application of calculus ideas, students can have a deeper understanding and grasp of dynamic motion problems in physics and improve their ability to solve practical problems [10].

The theory of AM is widely used in fields such as mechanical engineering, aerospace, and sports science. For example, flywheel design, attitude control of spacecraft, and analysis of athletes' rotational movements all involve the theory and calculation of AM [6].

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2.6. Application of calculus to two types of problems in particle kinematics

In particle kinematics, calculus is an important tool for analyzing and solving various motion problems. The main applications can be divided into the following two categories of problems: the relationship between position, velocity, and acceleration and Analysis of rate of change in motion.

- Relationship between position, velocity, and acceleration

In particle kinematics, the basic quantities that describe the motion of a particle include position, velocity, and acceleration. The relationship between these quantities can be established and analyzed through calculus.

- Position to velocity relationship

The position of the particle $s(t)$ It's time t Function, speed $v(t)$ is the derivative of position to time:

$$v(t) = \frac{d}{dt}s(t) \tag{35}$$

Case analysis: Assume that the motion position of a particle is given by the function $s(t) = 5t^2 + 2t$ We can get the velocity function by taking the derivative of the position function:

$$v(t) = \frac{d}{dt}(5t^2 + 2t) = 10t + 2 \tag{36}$$

- Velocity to acceleration relationship

Acceleration of a particle $a(t)$ is the time derivative of velocity:

$$a(t) = \frac{d}{dt}v(t) \tag{37}$$

Case analysis: Continuing with the above example, the speed function is $v(t) = 10t + 2$. Taking the derivative of the velocity function, we get the acceleration function:

$$a(t) = \frac{d}{dt}(10t + 2) = 10 \tag{38}$$

In this example, the acceleration is a constant, indicating that the particle is moving with uniform acceleration.

2.7. Analysis of the rate of change during exercise

In practical applications, the rate of change of particle motion (such as the change in acceleration and rate of change) usually needs to be analyzed through calculus. The following are two main types of applications:

- Displacement calculation for variable speed motion

For variable speed motion, the displacement can be found by integrating the velocity function. For example, in uniformly accelerated LM, the velocity function is:

$$v(t) = v_0 + at \tag{39}$$

Displacement $s(t)$ is the integral of the velocity function over time:

$$s(t) = s_0 + \int v(t)\,dt = s_0 + \int (v_0 + at)\,dt \tag{40}$$

The calculation results are:

$$s(t) = s_0 + v_0 t + \frac{1}{2} a t^2 \tag{41}$$

Case analysis: Assume that an object is in uniformly accelerated motion with an initial velocity of 4 m/s, the acceleration is 3 m/s^2, then the object is 5 The displacement after seconds is:

$$s(5) = s_0 + 4 \times 5 + \frac{1}{2} \times 3 \times 5^2 = s_0 + 20 + 37.5 = s_0 + 57.5 \tag{42}$$

2.8. Acceleration rate analysis

For the change of acceleration in accelerated motion, calculus can help analyze the change of acceleration over time. $a(t)$ is the derivative of velocity, while jerk is the derivative of acceleration concerning time: $\frac{d}{dt}a(t) =$ rate of acceleration change.

Case analysis: If the acceleration function is $a(t) = 2t$, then the rate of change of acceleration is:

$$\frac{d}{dt}(2t) = 2 \tag{43}$$

This indicates that the acceleration increases at a constant rate.

These applications demonstrate the importance of calculus in particle kinematics, which not only helps to establish the relationship between the quantities of motion but also solves complex motion problems through practical calculation and analysis. In the fields of physics and engineering, this analysis method is of great significance for predicting motion behavior and optimizing design.

3. Conclusion

In the study of particle kinematics, calculus, as a powerful mathematical tool, provides an important method for analyzing and solving motion problems.

Calculus has shown its potential for widespread application in dealing with variable speed motion and rate of change analysis. In variable speed motion, the changes in velocity and acceleration are integrated to obtain displacement and rate of change of acceleration, which provides an accurate tool for solving practical engineering problems. For example, calculating the displacement of a particle under variable acceleration conditions can effectively predict motion results and optimize design solutions. In the analysis of the rate of change of acceleration, calculus helps us understand how acceleration changes over time, further deepening our understanding of complex motion processes.

Calculus not only plays a key role in theoretical research but also demonstrates its important value in high school physics and practical applications. From basic kinematic formulas to complex variable speed analysis, calculus provides a systematic approach to solving various dynamic problems. Through the analysis and application of specific cases, students and engineers can more clearly understand and solve practical problems in particle motion and improve their ability to solve practical engineering challenges.

The application of calculus in particle kinematics not only enriches our understanding of motion phenomena but also provides a powerful mathematical tool for solving practical problems. The in-depth application of this method is of great significance to the development of physics and engineering.

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