Investigation of the effect of three parameters in the GEKO model on the calculation results for backward facing step flow

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Abstract. Computational fluid dynamics (CFD) is a field which is attracting more and more focus today for its ability to simulate comprehensive conditions. Reynolds-Averaged Navier-Stokes (RANS), which needs low cost, is one of the most popular methods used in engineering. However, RANS method has insufficient ability to simulate separated flow, which is a major challenge at present. One reason for this is that the model parameters introduce uncertainty into the simulation. This research investigates the effect of three parameters in the Generalized $k - \omega$ (GEKO) two-equation turbulence model on the calculations in backward facing step flow. It is found that C_{sep} has the greatest influence on the calculation of pressure coefficient and friction coefficient. C_{nw} has the greatest influence on the reattachment calculation through eddy-viscosity. The calculation results of the three variables show that there is correlation between them. Besides, a better set of parameters is obtained, which will improve the calculation accuracy for these three variables.

Keywords: Computational fluid dynamics, turbulence model, backward facing step, parametric analysis.

1. Introduction

With the rise of computers and increasing computational power, the field of Computational Fluid Dynamics (CFD) has become a common tool for predicting the real flow. CFD, is the process of using computational power to make mathematical predictions about physical fluid flow by solving the governing equations. In CFD methods used in present, Reynolds-Averaged Navier-Stokes (RANS) has become the most popular method in engineering for its low cost and acceptable accuracy of simulation.

During past decades, many researched has proposed lots of turbulence models to close RANS equations. For example, Launder proposed standard $k - \epsilon$ model in 1974[1] and Wilcox developed $k - \omega$ model in 1988[2], which are commonly used even today. However, the parameters in turbulence models introduce uncertainties into calculation, for the parameters are often set with several typical experiments, which can't represent all types of flow. Therefore, it is meaningful to explore the effect of parameters on calculations of flow, i.e. parametric uncertainty.

Research about parametric uncertainty can guide the calibration of turbulence models. Cheung[3] used Bayesian method to investigate parametric uncertainty of the S-A model, and calibrated the parameters with friction coefficients and velocity data, Chowdhary [4] discovered the inherent defect of the SST model when exploring parametric uncertainty based hypersonic turbulence flow experiment. These research all proved the necessity to learn about how parameters affect calculation results of

turbulence models. Therefore, this research focuses on the effect of the generalized $k - \omega$ (GEKO) two-equation turbulence model on backward facing step, which is a typical case to test the performance of turbulence model in simulating separated flow.

In general, this research will be divided into 4 sections. In the second part, methods used are illustrated. In Section 3, calculation details of the backward facing step are displayed. After completing the previous steps, the results will be analyzed. Finally, conclusion is drawn based on the results.

2. Method

2.1. Governing equations

There are three governing equations, which represent mass, momentum and energy conservation respectively:

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial}{\partial x_j}\left(\rho u_j\right) = 0 \\[2mm] \dfrac{\partial(\rho u_i)}{\partial t} + \dfrac{\partial}{\partial x_j}\left(\rho u_i u_j\right) = -\dfrac{\partial P}{\partial x_i} + \dfrac{\partial}{\partial x_j}\left(\tau_{ij} + \tau_{ij}^{\mathrm{t}}\right) \\[2mm] \dfrac{\partial(\rho E)}{\partial t} + \dfrac{\partial}{\partial x_j}\left(\rho H u_j\right) = \dfrac{\partial}{\partial x_j}\left[u_i\left(\tau_{ij} + \tau_{ij}^{\mathrm{t}}\right) + \left(\dfrac{\mu}{Pr_1} + \dfrac{\mu_{\mathrm{t}}}{Pr_{\mathrm{t}}}\right)\dfrac{\partial h_{\mathrm{s}}}{\partial x_j}\right] \end{cases}, \tag{1}$$

where ρ is the fluid density, u_i represents the velocity vector in x_i direction, P is the pressure, τ_{ij} and τ_{ij}^t are the molecular stress tensor component and the Reynolds stress tensor component respectively, μ and μ_t are the molecular viscosity and the turbulent eddy viscosity respectively, Pr_1 and Pr_t are the laminar and turbulent Prandtl number respectively, and h_s is the specific enthalpy. E and H are the total energy and enthalpy, which can be written as:

$$E = e + \frac{u_i u_i}{2}, \tag{2}$$

$$H = E + \frac{P}{\rho}, \tag{3}$$

where e is the specific internal energy.

2.2. The generalized $k - \omega$ two-equation turbulence model

The GEKO turbulence model is proposed by Mentor[5]. The model formulation is given by:

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial\left(\rho U_j k\right)}{\partial x_i} = -\tau_{ij}\frac{\partial U_i}{\partial x_j} - C_\mu \rho k \omega + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right], \tag{4}$$

$$\frac{\partial(\rho \omega)}{\partial t} + \frac{\partial\left(\rho U_j \omega\right)}{\partial x_j} = C_{\omega 1}F_1\frac{\omega}{k}P_k - C_{\omega 2}F_2\rho \omega^2 + \rho F_3 CD + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right], \tag{5}$$

with

$$\mu_t = \rho v_t = \rho \frac{k}{\max\left(\omega, \frac{S}{C_{Realize}}\right)}, \tag{6}$$

$$C_{Realize} = \frac{1}{\sqrt{3}}, \tag{7}$$

$$CD = \frac{2}{\sigma_\omega}\frac{1}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}. \tag{8}$$

The main characteristic of the GEKO model is that it has several additional parameters. These free parameters can be adjusted to cover a wide range of flow variations, and they are embedded through the functions F_1, F_2 and F_3 in the above equations. Parameters added in the GEKO model are designed to control specific physical aspects of the model, including near wall behavior, separation, free jet dynamics and so on. In this research, we selected three important parameters C_{sep}, C_{nw}, C_{jet}.

2.3. The prior range of the parameters
To ensure the convergence of calculations, we set the limit of the prior range of parameters as $\pm 25\%$ of the nominal value. The prior range of C_{sep}, C_{nw}, C_{jet} is listed in Table 1.

Table 1. The range of parameters of $k - \omega$ GEKO model

Parameters	C_{sep}	C_{nw}	C_{jet}
Lower	1.3125	0.25	0.25
Normal	1.75	0.5	0.5
Upper	2.1875	0.75	0.75

In the experiment, the Latin hypercube sampling is chosen to be the method to calculate data. Latin hypercube is a random sampling technique designed to reduce the correlation between input variables and thus improve the accuracy of Monte Carlo simulations. It ensures the randomness and homogeneity of the samples by dividing the range of values of each component equally into equal intervals and randomly selecting a value within each interval. The sampling number K can be calculated by[6]:

$$K = \frac{(n + p)!}{n!\, p!} \times n_p \tag{9}$$

where n is the number of parameters, p is the polynomial chaos number, and n_p is oversampling ratio. In this paper, with $n = 3$, $p = 2$ and $n_p = 2$, we can obtain $K = 20$.

3. Simulation details
As one of the simplest and most typical two-dimensional separated flow, the backward facing step[7] has been universally emphasized and tested. The geometry shape of the experiment is shown in Fig. 1, where $H = 1.27cm$ is the height of the step and U_{ref} is the reference velocity at the inlet.
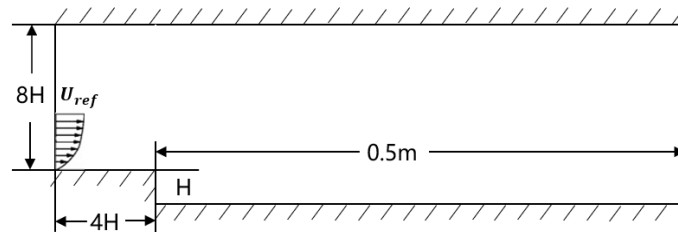

Figure 1. Geometry shape of the backward facing step

The grid will influence the accuracy of the simulation. To avoid the effect of grids, grid Reynolds number is set as $y^+ < 1$. The scheme of the grid is shown in Fig. 2.
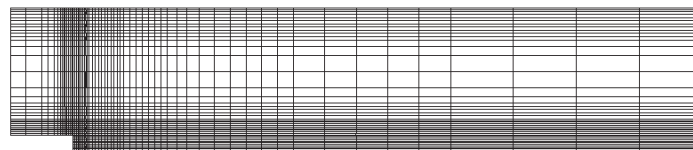

Figure 2. The scheme of the grid of the backward facing step

To ensure that the velocity distribution upstream the step is consistent with the experiment, the boundary type of the inlet is chosen as the velocity inlet. Detailed inlet conditions are shown in Table 2. The wall is set as adiabatic wall. The SIMPLEC algorithm is used, the second-order upwind format is adopted. Calculations are conducted with Fluent. All simulations are converged.

Table 2. Inlet conditions of the simulation

Variables	Value
Reference velocity U_{ref}	44.2m/s
Mach number	0.128
Pressure	101325Pa
Temperature	288.15K
Boundary layer thickness	1.9cm

4. Analysis and discussion

4.1. Normal calculation

Firstly, the calculation is done with nominal values of the parameters. The contour of the velocity is shown in Fig. 3. It can be seen that the boundary layer starts to separate at the edge of the step, and backflow occurs in the separation area downstream of the step. The separation area is mainly concentrated in the triangle area from the step to the reattachment point, and the boundary of the separation area is clear. Then the boundary layer continues to develop in the downstream boundary layer of the separation zone until the exit.
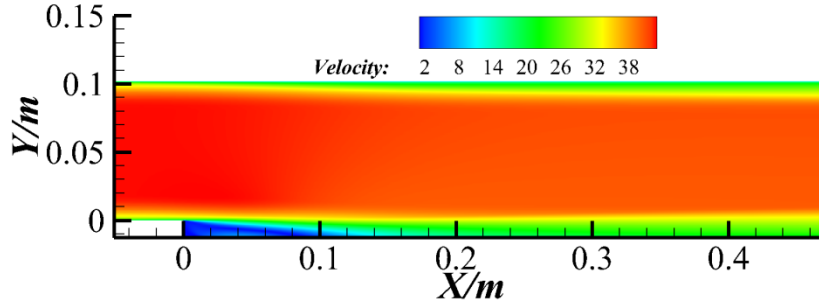

Figure 3. The contour of the velocity for backward facing step with nominal parameters

4.2. Prior analysis

Prior results calculated with 20 sets of parameters are shown in Fig. 4. It can be seen that the results obtained by using the nominal values of the parameters are within the prior range. The pressure coefficient of the separation zone, recovery zone and development zone has little change with the original parameter value after the change of parameters, and generally cannot cover the experimental value, which means that the change of three parameters cannot make up for the defects of the model itself. For the friction coefficient, changing the parameters' values significantly affects the calculation results. More experimental points are covered, indicating that calibrating the model parameters may improve the friction coefficient better than the pressure coefficient.
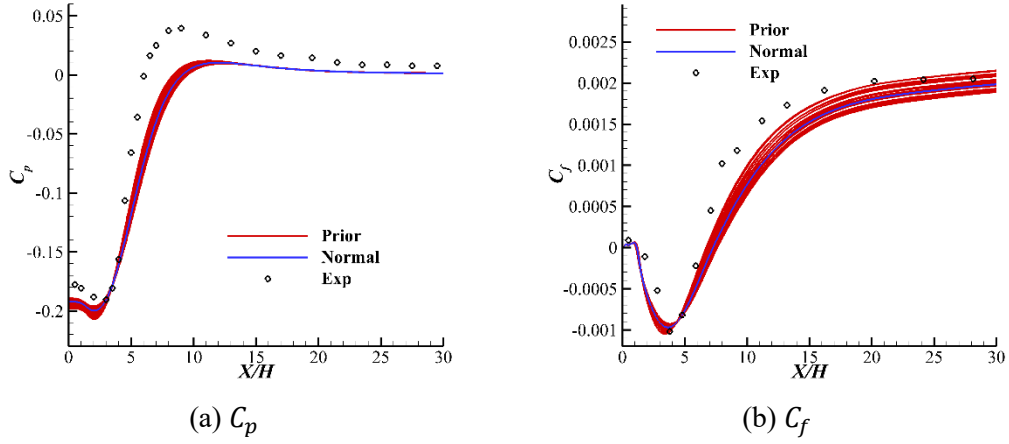
(a) C_p (b) C_f

Figure 4. Prior results calculated with 20 set of parameters

We can obtain the sensitivity index of each parameter by constructing polynomials, that is, the Sobol index. We can decompose a variable into a sum of polynomials as

$$\alpha^*(X,\xi) \approx \sum_{j=0}^{P} \alpha_j(X)\Psi_j(\xi), \tag{10}$$

where $\alpha^*(X,\xi)$ is the value of the variable calculated with the parameter ξ at location of X. The mean μ and variance D of the variable can be given by

$$\mu = \alpha_0(x),$$
$$D = \sum_{i=1}^{P} \alpha_i^2(x)\langle \Psi_i^2(\xi)\rangle. \tag{11}$$

According to Crestaux et al.[8], D can be split into

$$\begin{cases} D = \sum_{i=1}^{i=n} D_i + \sum_{1\leq i<j\leq n}^{i=n-1} D_{i,j} + \sum_{1\leq i<j<k\leq n}^{i=n-2} D_{i,j,k} + \cdots + D_{1,2,\ldots,n}, \\ D_{i_1,\ldots,i_s} = \sum_{\beta\in(i_1,\ldots,i_s)} \alpha_\beta^2\langle \Psi_\beta^2(\xi)\rangle, 1\leq i < \cdots < i_s \leq n. \end{cases} \tag{12}$$

Then the Sobol index can be given by

$$S_{i_1\ldots i_s} = \frac{D_{i_1\ldots i_s}}{D}. \tag{13}$$

The Sobol indices of three parameters for the pressure coefficient and friction coefficient are shown in Fig. 5. As can be seen, the sensitivity of C_{sep} dominates whether the results of C_p and C_f, which clearly proved the important effect of the construction of C_{sep}. For C_p, in the separation zone, the Sobol indices of C_{nw} and C_{jet} is extremely low. While after the reattachment point, the sensitivity of C_{nw} and C_{jet} begin to increase. For C_f, the sensitivity is chaos near the step, which is caused by the appearance of the corner vortex. In other region, the sensitivity of C_{jet} maintains low, while the sensitivity of C_{nw} recovers downstream.
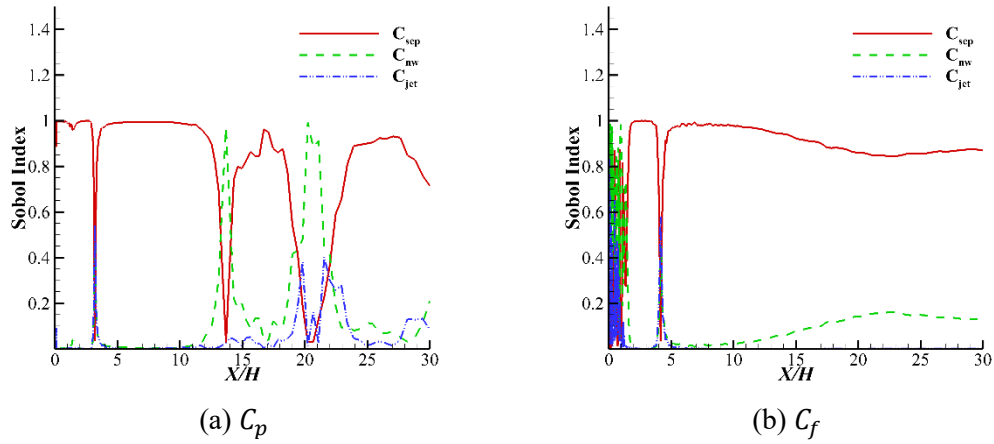
(a) C_p (b) C_f

Figure 5. Sobol indices of three parameters for C_p and C_f

The Sobol indices of three parameters for the location of reattachment point are shown in Fig. 6. C_{nw} is the most sensitive parameter and the sensitivity of C_{sep} and C_{jet} are similar. This is not very surprising because increasing C_{nw} leads to higher wall shear stress in non-equilibrium flows, which will directly affect the location of the reattachment point.
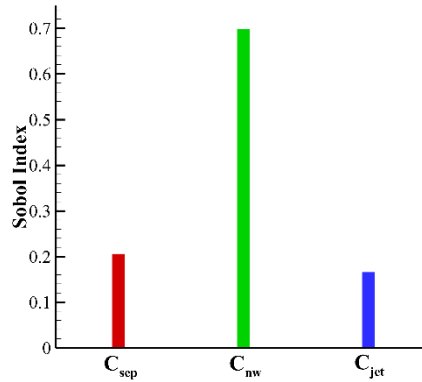

Figure 6. Sobol indices of three parameters for the location of reattachment point

The values of the parameters and the reattachment point are listed in Table 3. From the table. We can see that there are 9 sets of calculations which give less relative error of the reattachment point than with nominal values of the parameters. For these samples, the denominator is that they all have small C_{sep}, which means that although the Sobol index of C_{sep} is not the biggest, it plays an important role in the process of reattachment. By decreasing C_{sep}, eddy-viscosity is decreased too, which leads to more sensitivity to adverse pressure gradients for boundary layers and lower spreading rates for free shear flows. This results in a smaller separation zone.

Table 3. The values of the parameters and the reattachment point

Set	C_{sep}	C_{nw}	C_{jet}	Reattachment point (X/H)	Relative error
Normal	1.75	0.5	1.5	7.24	18.63%
1	2.1875	0.54625	0.54625	7.61	24.82%

Table 3. (continued).

2	1.728125	0.52	0.48	7.28	19.27%
3	1.404375	0.4275	0.415	6.77	11.01%
4	2.0475	0.59875	0.4675	7.49	22.76%
5	1.588125	0.50625	0.585	7.07	15.92%
6	1.360625	0.5725	0.49375	6.82	11.79%
7	1.82	0.585	0.55875	7.32	20.05%
8	1.3125	0.4675	0.52	6.69	9.72%
9	1.86375	0.55875	0.375	7.36	20.69%
10	1.49625	0.5325	0.40125	7.02	15.14%
11	1.771875	0.45375	0.38875	7.28	19.27%
12	2.095625	0.49375	0.4275	7.53	23.40%
13	1.4525	0.61125	0.59875	6.86	12.43%
14	1.63625	0.415	0.50625	7.15	17.21%
15	2.139375	0.44125	0.5325	7.61	24.82%
16	2.00375	0.38875	0.44125	7.49	22.76%
17	1.68	0.625	0.45375	7.20	17.98%
18	1.544375	0.40125	0.61125	7.02	15.14%
19	1.955625	0.48	0.625	7.49	22.76%
20	1.911875	0.375	0.5725	7.45	22.11%

The relative error calculated with 20 sets of parameters for C_p and C_f is listed in Table 4. We can see that the calculation of the reattachment point is positively correlated with the calculation of C_p and C_f to some extent. That is to say, the samples with a better calculated reattachment point also have better calculated C_p and C_f, although this doesn't mean that better C_p will lead to better C_f. Besides, a better calculated reattachment point tends to better C_f. This is because the correlation between the reattachment point and C_f: The reattachment point is the location where C_f equals to zero. All in all, considering the three variables we obtained, $C_{sep} = 1.3125$, $C_{nw} = 0.4675$ and $C_{jet} = 0.52$ (Set 8) lead the best results.

Table 4. The relative error calculated with 20 sets of parameters for C_p and C_f

Set	Relative error (C_p)	Relative error (C_f)
Normal	36.44%	29.08%
1	40.77%	34.60%
2	36.98%	28.95%
3	31.48%	23.05%
4	40.01%	33.04%
5	34.78%	26.45%
6	30.12%	21.56%
7	37.88%	30.11%
8	29.12%	20.97%
9	38.62%	30.91%
10	33.26%	24.56%
11	37.72%	29.95%
12	40.50%	33.95%
13	32.04%	23.30%

Table 4. (continued).

14	35.79%	27.78%
15	40.64%	34.54%
16	39.96%	33.34%
17	36.20%	27.72%
18	34.15%	26.04%
19	39.17%	32.32%
20	39.02%	32.21%

5. Conclusions

In a nutshell, we conduct calculations with different values of parameters of the GEKO turbulence model based on backward facing step flow. With Latin hypercube sampling, parameters are obtained in the form of a uniform distribution. Then the prior range of the pressure coefficient and friction coefficient and the sensitivity of parameters is analyzed. At the same time, the calculation error of the reattachment point is compared. Some conclusions are drawn:

1. C_{sep} plays an important role in the calculations of separation, especially for the pressure coefficient and friction coefficient. C_{nw} dominates the simulation of the reattachment point, which is because C_{nw} directly affects eddy-viscosity.

2.   The calculation of the reattachment point, C_p and C_f appears to be correlated that a better reattachment point will lead to better calculated C_p and C_f. Better C_p doesn't mean better C_f, but a better reattachment point often corresponds to better C_f.

3.   A smaller C_{sep} will lead to a smaller separation zone, which results in the increase of calculation accuracy for backward facing step. $C_{sep} = 1.3125$, $C_{nw} = 0.4675$ and $C_{jet} = 0.52$ (Set 8) will lead to a better result for backward facing step.

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