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Research on the Application of Mathematics in Finance

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Abstract. Mathematics is a fundamental science and a problem-solving method that plays an important role in revealing the essential laws of the development of certain things. It has rigorous logic, high abstractness, and wide applicability. This brings great difficulties to enterprises, especially high school mathematics because it involves a wide range of knowledge points and complex calculations. Among them, the formation of financial mathematics has greatly promoted the development of China's financial industry. Moreover, in solving financial problems, relevant knowledge and theories of mathematics can be used to analyze and explore financial laws, and the application effect is significant, for example, using deterministic mathematical method and nondeterministic mathematical method to solving question about financial investment and returns; building securities investment portfolio model or asset valuation model to calculating and expressing some economic models. Based on this, this article discusses the application of mathematical knowledge to several financial problems, analyzes the current problems, and looks forward to the future of financial mathematics.

Keywords: Financial Mathematics, financial investment, Asset Pricing, option valuation.

1. Introduction

In the late 20th century, finance increasingly showed a fusion with mathematics: on the one hand, using appropriate mathematical methods to analyze and solve financial problems. On the other hand, the constantly emerging real-world problems in finance have also provided valuable theoretical research directions for mathematics and statistics. This reality has led to the gradual formation of a new interdisciplinary field between finance and mathematics, which has gradually developed into a new discipline - financial mathematics. Its main feature is to use effective mathematical tools and methods to reveal the essential characteristics of finance and the universal laws of economic operation, and to analyze and study the reasonable pricing and risk avoidance strategies of various uncertain claims with potential risks, in order to solve problems in the financial field. The concepts of arbitrage, optimality, and equilibrium involved in it have made the development of the financial industry more and more regular. On the other hand, the development of finance has also enriched the branches of mathematics. Below, this paper will conduct a comprehensive study on the application and innovative improvement of mathematical methods in financial investment based on experience, in order to find a basis in financial theory to help the development of China's financial industry.

2. Translation of Theoretical Framework and Main Research Issues of Financial Mathematics

In the process of financial investment, there are many mathematical problems involved, which makes financial investment a collection of mathematical methods. In terms of logicality, financial investment

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has stronger logicality, which exacerbates its abstract characteristics. However, mathematical methods can transform abstraction into more straightforward numerical problems, effectively overcoming the complex quantitative relationships in financial investment problems. Ultimately, we can achieve a comprehensive analysis of financial investment problems and fully express their logical relationships through digital forms. Specifically, financial mathematics mainly uses modern mathematical tools such as stochastic analysis, stochastic control, mathematical programming, differential games, nonlinear analysis, mathematical statistics, functional analysis, martingale theory, backward stochastic differential equations, fractal geometry, etc. to study the following main issues: firstly, how to combine investment securities to obtain maximum returns or reduce investment risks; secondly, Capital asset pricing models and optimal investment and consumption theories for incomplete financial market securities (such as futures, options, and other derivative instruments); thirdly, The term structure of interest rates and the pricing theory of interest rate derivatives; finally, Risk management and risk control theory for incomplete financial markets. Some people have also introduced new nonlinear analysis tools in securities price analysis, such as fractal geometry, chaos theory, wavelet analysis, pattern recognition, etc. Some people use neural network methods and artificial intelligence methods in securities selection and stock type prediction, while others use simulated annealing and genetic algorithms in simulation research on futures market innovation [1, 2].

In summary, several important theoretical frameworks in the field of financial mathematics are: modern portfolio theory, capital asset pricing model, arbitrage pricing theory, hedging theory, option pricing theory, interest rate term structure theory, etc.

3. The Application of Common Mathematical Knowledge to Financial Problems

3.1. The application of mathematical knowledge to financial investment and returns

In actual financial investment activities, financial risk and return are the two core forms, and there is a complementary relationship between these two forms. Meanwhile, risk and return are relative, and reducing the risk of financial investment does not necessarily mean achieving returns. Generally speaking, the risks and returns in the process of financial investment are mainly influenced by factors such as interest rates, commodity prices, and stock market trading prices. At present, the application of mathematical knowledge to financial investment and returns is mainly based on mathematical methods for measuring financial risks. There are two commonly used methods: deterministic mathematical methods and non-deterministic mathematical methods.

3.1.1. Deterministic mathematical method. Traditional risk analysis is mainly based on bond prices, bond yields, stock prices, and stock indices. Financial researchers calculate and analyze mathematical indicators that affect the risk status of financial investment activities before selecting financial investment portfolios, and then use them as a basis for trading activities. However, with the continuous complexity of the financial market, these statements are not rigorous enough in terms of the degree of concern. Deterministic mathematical methods can abstract these factors and indicators into deterministic mathematical variables by studying and analyzing the various constituent factors and evaluation indicators of financial investment risks. At the same time, basic mathematics can be used to comprehensively analyze and calculate the data, analyze the risk coefficient of actual investment in the later stage, and provide theoretical support for actual investment in the later stage. Based on the analysis results, we can make reasonable adjustments to the financial investments that have been implemented or are about to be implemented, gradually achieving the maximization of the benefits of financial investments. Generally speaking, we need to perform relevant calculations on parameter object indicators. The first step is to set up a new bond yield indicator S, with the annual yield level on the bond face set to r, the bond face value to M, the bond issuance price to N, and the bond repayment period to T. Based on these indicators, we will implement the application formula for the yield to maturity of newly issued bonds. With the help of preliminary analysis results and actual market trends, we can use

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deterministic mathematical methods to provide the highest quality activity plans for later actual investments[3, 4].

3.1.2. Non-deterministic mathematical method. In the field of financial investment, the factors that lead to risks in financial investment activities are not the only ones, and there may be many uncertain factors involved. Unstable factors can lead to comprehensive dynamic changes in financial investment, and based on this, we cannot solve it solely through deterministic mathematical methods. This requires us to introduce non-deterministic mathematical methods to address the impact of uncertain factors. In the development of financial mathematics, there are many types of non-deterministic methods, such as probability theory, mathematical statistics, stochastic processes, etc. These mathematical methods can be fully combined with the financial investment process. The current non-deterministic mathematical methods mainly convert the economic losses and returns that investors may suffer from actual financial investment activities into random mathematical variables. After the conversion is completed, specific data objects can be calculated, processed, and analyzed using statistical data calculation methods such as mathematical variance, expectation, and standard deviation in the discipline of mathematical statistics. This process is also the most effective way for non-deterministic mathematical methods to control financial investment risks in financial trading activities[3, 4].

3.2. The Application of Mathematical Methods in Financial Forecasting and Decision making

There are many uncertain factors in the actual operation of financial markets, especially in financial activities. Investors need to predict and analyze future financial variables such as the coverage rate, savings deposit balance, inflation rate, etc. before investing. The commonly used mathematical methods in financial forecasting include the least squares method, the first and second moving average method, the first and second exponential smoothing method, the modified exponential curve method, the growth curve prediction method, the univariate linear regression method, three-point method, the Markov prediction method, Kalman filter method, the two-step prediction method, etc. The commonly used mathematical methods in financial decision-making include the extreme value selection decision method, the linear programming decision method, the expected value method, the marginal analysis method, the indifference curve method, the maximum output combination method, the minimum cost combination method, etc. [5].

4. The Application of Mathematical Models in Financial Theory

As a branch of economics, finance still draws energy from economics in the field of theoretical applications of finance. This includes the use of economic models, but economic models cannot exist without the relationship between numbers and quantities. Here, mathematical thinking and mathematical relationships need to be applied, and financial mathematics can help us accurately calculate and express some economic models. This includes: the securities investment portfolio model, the capital asset pricing model, the option pricing equation, etc. This paper takes the securities investment portfolio model and asset valuation model as examples to illustrate them.

4.1. Securities investment portfolio model

When calculating the securities investment portfolio model, a part of it is the calculation of the returns in the asset portfolio, which requires the application of mathematical thinking and methods. When calculating, we can set the securities prices in the investment portfolio as a random variable. By using the mean of this variable to represent returns, the expected return E (rp) in the investment portfolio can be calculated. E (rp) is the average expected return of all assets included in the investment portfolio. We can use the following formula to calculate the expected return:

$$E(rp) = E(xr1 + xr2 + xr3 + \cdots + xrn) = x1E(r1) + x2E(r2) + \cdots + xnE(rn) \quad (1)$$

$$E(rp) = \sum_{i=1}^{n} x1E(r1) \quad (2)$$

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Where $x1 + x2 + x3 + \ldots + xn = 1$

Ri is the expected return type I, and i is the discount rat [6].

4.2. Asset valuation model

Renowned American economist Irving Fischer proposed the theory of asset valuation in 1986. He believes that the current value of assets should be equal to the sum of all discounted values that maintain cash flows. This theory has become the foundation for establishing asset valuation models in modern financial theory. The foundation of this theory is based on the timeliness of funds, which have a time value. In the vertical time dimension, the value of funds varies at different times. This also requires this study to not simply accumulate or subtract funds in calculations. There is more than one asset valuation model, and the simplest one is to replicate the formula. In this valuation model, we use the following mathematical method for calculation:

Assuming the cash flow of the investment at a future time t is C(t), the discount rate is R(t), the number of periods is n, and the total present value is PV, we can obtain:

$$PV = \sum_{i=1}^{n} C(t)[1 + R(t)]^{-1} \tag{3}$$

This mathematical formula laid the foundation for estimating the capital changes in securities investment value [6].

5. Conclusion

In summary, mathematical methods run through the entire process of financial investment, which is also an important foundation for improving the level of financial investment. In our practical research on financial investment, mathematics, and finance can be organically combined, thus forming the concept of financial mathematics. The application of financial mathematics can not only help investors analyze risk factors to reduce the potential loss of investment benefits, but also make relevant financial predictions to help financial market trading activities achieve optimal activity goals, thereby obtaining maximum benefits and promoting the healthy development of the financial industry. Along with the rapid development of financial mathematics, there are also some problems, such as the assumption that financial market mathematical models are based on rational expectations, assuming that market participants (institutions or individuals) can rationally utilize and process all available information, except for random factors, and that people have complete foresight. But in reality, the expectations of financial markets are irrational, and short-term expectations tend to be influenced by speculation. In addition, the mathematical models of financial markets are established within the framework of neoclassical economics theory and are based on market equilibrium models and linear mathematical methods. However, in practice, the equilibrium of the financial market is short-lived, and the imbalance is absolute. Furthermore, the conclusions obtained through mathematical models or quantitative analysis methods are semi empirical, semi rational, and so on. The existence of these problems causes deviations between the theoretical calculations of mathematical models and actual prices. Although they fit historical data well, they do not have a high success rate for future predictions. It can be foreseen that the application of mathematics in the financial field should be based on existing theoretical frameworks, enriching the methods and results of theoretical research, so that mathematical models can better explain reality. In addition, a combination of qualitative and quantitative analysis should be conducted from various aspects such as fundamentals, technology, and information, starting from the overall financial market. Only then can the established mathematical model reflect the essence of the financial market system as much as possible and play an optimization role. Meanwhile, the study of mathematical knowledge has become a major area of research in finance. Mathematics is constantly driving the development of financial practice. Therefore, mathematics has broad application prospects in the financial market.

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