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The Game Theory of Football Penalty Kicks

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Abstract. Soccer is the world's number one sport, known for its fierce rivalries and full of surprises. Especially in important international competitions, such as the World Cup and the European Championships, matches are often about national honor and the expectations of millions of fans. Penalty kicks are a common punishment in soccer, where the goalkeeper and the shooter are pitted against each other in a one-on-one confrontation. Penalty kicks, because of their simplicity of confrontation, have a breath-taking tension and excitement, and most of the victory depends on the decision-making confrontation between the shooter and the goalkeeper. Game theory is an important tool for studying how participants make decisions to maximize their own gains in situations of conflict and competition. In order to study how the goalkeeper saves most effectively, this paper analyzes the penalty shot from the perspective of game theory, and obtains the optimal solution for the goalkeeper's save direction in a simplified situation by calculating the Nash equilibrium under mixed strategies.

Keywords: game theory, football penalty kick, Nash equilibrium.

1. Introduction

With the ending of the European Cup 2024 in a month, there are many impressive moments in this event and a lot of penalty kicks in the races. Before starting the game and the penalty kick, the coach of each team would encourage the players and make an order for the penalty kicks. Also doing some calculations to guess which direction the goalkeeper will choose. There are some connections between game theory and penalty kicks. Using the modeling of penalty kicks in football can help us better explain the connection, while these circumstances do not only happen in football, these also happen in other sports games like baseball which kind of ball the stock bowler will shoot and which ball the batsman will hit.

Game theory is a branch of economics that analyzes an individual's optimal game strategy by examining the decisions that can be taken by different individuals in a game and the responses they make to the outcomes. In penalty kicks, the goalie and the shooter are engaged in what is clearly a zero-sum game, which we can thus analyze using game theory [1].

This paper focuses on how game theory can be used to analyze decision-making in penalty shot confrontations. Mathematical modeling of the penalty kick confrontation was carried out using mixed strategies in game theory, and we came up with the optimal strategy for the goalkeeper to make a save decision. Mixed-strategy Nash equilibrium is a commonly used concept in game theory, which is shown to be usable in-game analysis for free throws and is shown to fit well with real-world data [2-3]. This

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paper is a guide for goalkeeper decision-making in soccer. An initial simple case study is also provided for the modeling analysis of penalty kicks using game theory.

2. Football penalty kick rules and the possibilities of the direction the player and goalkeeper chooses

There are several assumptions for us to take. Firstly, The players can only aim the ball to either the left, the right and the middle and the goalkeeper can only dive either the left, the right and the middle as well. Secondly, the speed of the ball is suitable enough for the players and goalkeepers to do the reaction at actually the same time and the ball is shot in the door frame range, so we can make the conclusion that there are totally two possibilities: one is the player scores, the other is the goalkeeper succeeded diving the ball.

3. Some concepts of Game theory

3.1. Nash equilibrium

Nash Equilibrium is a concept in game theory that describes the optimal combination of strategies for each player in a game. In a Nash equilibrium, each player knows the strategies of the other players, and no one player can achieve a better outcome by unilaterally changing his or her own strategy. In other words, a Nash equilibrium is a policy configuration in which each player's strategy is the best response to the other player's strategy. Specifically, if a player in a Nash equilibrium changes his or her strategy to make his or her return worse, then the state is in equilibrium.

3.2. Mixed strategy

Mixed strategy is a concept in game theory used to describe a game in which players can choose their strategies with a certain probability distribution. Unlike pure strategies, which choose the same definite strategy every time, mixed strategies allow participants to randomize their choices. Specifically, the core of the mixed strategy is the probability distribution in which each participant assigns a probability to each possible strategy, representing the likelihood that they will choose that strategy. Equilibrium state: In a mixed strategy Nash equilibrium, each player chooses his or her own mix of strategies, making it impossible for other players to increase their expected returns by changing their own strategies when they know the mix of strategies. Mixed strategies allow participants to avoid being predicted and exploited by their opponents when facing uncertain or competitive situations.

3.3. Pure strategy

Pure strategy is a basic concept in game theory that refers to the fact that players choose the same definite strategy each time in a game. In a pure strategy, there is no randomness or uncertainty in the participants' actions, and their choices are fixed and predictable. Specifically, the characteristics of a pure strategy include certainty and no randomization. Certainty means each player adopts a fixed strategy in each case, that is, in the same game environment, their strategy choices are consistent. No randomization refers to unlike mixed strategies, pure strategies do not involve a probability distribution or random selection.

4. Connection between game theory and penalty kick

From the sight of the shooter, there are three possibilities: left, middle and right(for the goalkeeper, middle stands for stay rest). The payoff for the player is to score and the payoff for the goalkeeper is that the player does not score. The payoff matrix is shown below(see Table 1):

Table 1. Payoff matrix of goalkeeper and shooter

Payoff matrix		Goal keeper	Goal keeper	Goal keeper
		left	middle	right
Shooter	left	0.65,0.35	0.95,0.0.5	0.95,0.05

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Table 1. (continued).

Shooter	middle	0.95,0.05	0,1	0.95,0.05
Shooter	right	0.95,0.05	0.95,0.05	0.65,0.35

Simply, there is a connection between the probability of winning and the actual performance. If the goalkeeper judges incorrectly, for example, the player shoots left and the goalkeeper dives right, the probability for the shooter to score is about 95%. But if the goalkeeper judges correctly, the probability of the shooter failing to score is 35%. There is no great probability that both two of them choose middle and we assume that the shooter cannot score in this circumstance.

Obviously, there is no Nash equilibrium in this game. For example, if the goalkeeper always tends to choose right in the penalty, then Ronaldo will tend to shoot middle or left. If the goalkeeper is expected to choose right, Ronaldo will not shoot in right. Table 2 shows the probabilities for the different consequences between the shooter and the goalkeeper. As expected, both of them tend to make their choice more ambiguous, because unpredictability is the key to winning.

Table 2. Probability distribution of decision combinations in penalty kicking

Taker	Keeper	Probability(%)
Left	left	19.6
Left	middle	0.9
Left	right	21.9
Middle	right	3.6
Middle	middle	0.3
Middle	right	3.6
Right	left	21.7
Right	middle	0.5
Right	right	27.6

According to the mixed strategy from the shooter, the probability of choosing left which uses Sl as a symbol, then the same as Sm as middle and Sr as right. So we can get that Sl + Sm + Sr =1, Sm = 1 - Sr - Sl, for different value of Sl and Sr, the shooter has a different mixed strategy. Similarly, using Gl, Gm and Gr as the symbol of the probability for the goalkeeper to choose left, middle and right, and Gm = 1 - Gl - Gr, now the values of Gl and Gr decide the mixed strategy of the goalkeeper.

A Nash equilibrium includes two random strategies (left and right), or even all three choices are randomized. We can assume that for every player a Nash equilibrium where all three pure probabilities are positive. Above all, the expected payoff for the shooter to shoot left, middle and right are the same. There are the different expected payoffs below in different situations:

payoff for shooter to choose left:

$0.65 \times Gl + (1 - Gl - Gr) \times 0.95 + Gr \times 0.95 = 0.95 - 0.3Gl$ (7.5)

payoff for shooter to choose middle:

$0.95 \times Gl + (1 - Gl - Gr) \times 0 + Gr \times 0.95 = 0.95(Gl + Gr)$ (7.6)

payoff for shooter to choose right:

$0.95 \times Gl + (1 - Gl - Gr) \times 0.95 + Gr \times 0.65 = 0.95 - 0.3Gr$ (7.7)

We want to find a strategy for the goalkeeper: to evaluate the value of Gl and Gr, and now all the expected payoff for the shooter is equal. Find the equation(7.5) and(7.7), the expected payoff for us to choose left and right are the same:

$0.95 - 0.3Gr = 0.95 - 0.3 Gr \Rightarrow Gl = Gr$

So in Nash equilibrium, the probability produced for the goalkeeper to choose left and right are the same. In other words, if the goalkeeper chooses left or right evenly, then the probability for the goalkeeper to choose left and right is the same. Using the symbol G to represent this equal value: Gl = Gr

= G. In equation(7.5) to(7.7), using G to replace Gl and Gr, we can get that the payoff for the shooter to choose left or right turns to 0.95 - 0.3g, while the probability of choosing middle is 1.9g. The shooter expects the three choices to have the same probability. Then we can make a conclusion:

0.95 - 0.3 G = 1.9 G

G = 0.952.2 = 0.43

In conclusion, Nash equilibrium makes the probability of the goalkeeper choosing left is 0.43, and for right is 0.43 as well, while staying in middle is only 0.14. Only by satisfying these probability conditions, the goalkeeper will uniformly make his decision.

Of course, the goalkeeper finds that only randomizing his decision can make his choice in the best way, every choice requires the same expected payoff. The same deduction can deduce the strategy of the shooter to make the goalkeeper for his decision more evenly. From the content above, we can get that, the shooter who shoots left has a probability of 0.43, who shoots right has a probability of 0.43, shoots middle has a probability of 0.14. With these uniform strategy, the probability to score can be calculated:

$$0.43 \times (0.43 \times 0.65 + 0.14 \times 0.95 + 0.43 \times 0.95) + 0.14 \times (0.43 \times 0.95 + 0.14 \times 0 + 0.43 \times 0.95) + 0.43 \times (0.43 \times 0.95 + 0.14 \times 0.95 + 0.43 \times 0.65) = 0.82044.$$

The first term stands for the probability of the goalkeeper diving in the direction of right(0.43). In this situation, shooter can choose to right (with probability 0.43), then the probability to score is 0.65. If shooter shoots to middle(0.14), then the probability to score is 0.95. If shooter chooses left(0.43), then the probability to score is 0.95. The second term and the third term stands for the goalkeeper to dive in the direction of middle and left, so we can make a rough estimation that the probability is 0.82.

For this theory, another guess in multiple choices the probability to score is the same. Only if the probability of scoring is the same, the shooter is more pleasure to shoot in direction more randomized, even left or right is suitable. An experiment supports the equalization theory: for 22 different players, according to shooter natural shooting direction, the probability to score is 0.8268. Natural shooting direction means that a player gets used to shoot left would shoot left or middle , while a player gets used to shoot in direction of right would shoot in right or middle. While anti natural shooting direction(for example, player who gets used to shoots right shoots to left) has the probability to score is 0.8111. From these two data, the difference between 0.8268 and 0.8111 is really small. So as expected, the probability to shoot in both direction is the same.

5. Further investigations

What we have presented in the article is just a simplified version of the model to explore how game theory can be applied to analyze penalty kicks. However, penalties involve a very large number of influencing factors, e.g., the players themselves are characterized, such as habitual foot and habitual shot direction, which have been shown to be useful in predicting the direction of the shot, thus helping the goalkeeper in his decision making [4]. Numerous cases and data have proved that even top players have fixed kicking habits and thus may be used to develop targeted defensive strategies [5]. In addition to a player's own motor habits, the importance of the game and the conditions of the game may also have an impact on a player's shooting form and decision-making [6].

6. Conclusion

This paper focuses on how to use mixed-strategy Nash equilibria to analyze games in penalty kicks. Overall, mixed-strategy Nash equilibrium can be used to analyze the game between the goalkeeper and the shooter in penalty kicks, and we have computed the optimal strategy of the goalkeeper in the simplified case. However, the possible influences in the real world are much more complex than the simplified case, and we can make the predicted model more realistic and practical by adding more possible factors. Statistical data can be used to optimize our estimation of the probability of scoring a goal, making the calculation more consistent with the real situation. A further breakdown of the direction of shooting and pouncing can lead to richer decision-making options for goalkeepers and shooters, and will make the game situation more complex. These are all possible directions of

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improvement, but due to the lack of relevant data, we did not carry out such improvement and analysis. Future research can combine the theoretical model and actual data to get more realistic research results with more practical significance.

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