Effect of Sex Ratio in Lamprey on Resource Availability and Ecosystem

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Abstract. Lamprey populations significantly affect ecosystems and can adjust their sex ratio based on environmental conditions. This study examines lamprey population size and sex ratio by developing a single-species logistic growth model that incorporates corrections for gender structure and environmental resources. We analyze population changes with and without sex ratio adjustment under varying environmental conditions. To assess the ecological impact, we also create a multi-species competition model based on the Lotka-Volterra predator equation, evaluating the effects of sex-regulated lamprey populations on the food chain. The stability of both models is analyzed using allowable error inequalities and inverse distance functions, and we assess population changes through data volatility analysis.

Keywords: lamprey, species sex ratio regulation, system dynamics model, Lotka-Volterra predator equation.

1. Introduction

Most animal species are either male or female, though some exhibit varying sex ratios based on environmental conditions [1,2]. Sea lampreys, which inhabit temperate coastal and freshwater regions, show a sex ratio linked to environmental resources. Poor conditions lead to a male-dominated population (up to 78%), while good conditions increase females to about 56% [1].

Through in-depth analysis and research on the background of the problem, combined with relevant knowledge [2], we established a mathematical model that explores the impact of a lampreys population on a larger ecosystem when it can alter its sex ratio. And this has led to an exploration of the advantages and disadvantages faced by lampreys from multiple perspectives in order to gain a deeper understanding of the survival mechanisms of lampreys. We also explored the effects of lampreys populations on ecosystem stability when they can alter their sex ratio. When the sex ratio of lampreys can be altered, we analyzed the effect it will have on other populations in the ecosystem.

The study focuses on sex ratio differences in sea lampreys and their dependence on local conditions. A single-species population competition model and a multi-species competition model were established, according to which the interrelationships between sea lampreys and the environment were described in terms of male ratio, number of populations, resource availability and reproductive success, respectively.

2. Materials and Methods

2.1. Single - species population competition model

We know that lampreys are usually male or female. The proportions of females and males in the population are set to be $F(t)$ and $M(t)$, then:

$$M(t) + F(t) = 1 \tag{1}$$

Resource availability $R \in (0,1)$ represents the percentage of resources available to lampreys in the ecosystem in which they live. When R→0, resources are scarce, leading to higher mortality and a decrease in population size; when R→1, resources are abundant, promoting reproduction and an increase in population size. Therefore, it is reasonable to believe that when R= 0.5, the population size remains stable. The threshold R_{th} represents this balance [3]. Therefore, the change in the proportion of male lampreys can be described as:

$$\frac{dM}{dt} = -\alpha(R(t) - R_{th}) \tag{2}$$

Where α stands correlation coefficient between lamprey male sex change dM/dt and resource availability R.

This model only analyzes the lamprey species. We categorize other ecosystem elements affecting lamprey development as resource availability in R to simplify the model. The growth rate is proportional to the population size. Divide time into years. Based on the logistic model, the population N can be described as:

$$\frac{dN}{dt} = r \cdot N(t) \cdot \left(1 - \frac{N(t)}{K}\right) \tag{3}$$

Among them, r represents the inherent growth rate of the population, we assume that it is a constant from assumption 1, represents the ratio of population size to maximum capacity.

In fact, as the population size increases toward its maximum capacity, available resources R decrease and the growth rate r gradually diminishes, approaching 0. A coefficient β describes the relationship between population size and resources. In addition, sexual competition between lampreys, such as two males pursuing a female, increases mortality. The effect of sexual competition is represented by coefficient γ. A balanced sex ratio is considered when the male-female ratio is 1:1, with a threshold of 0.5. Thus, the absolute value of $M - 0.5$ represents the deviation of the current sex ratio from the balanced sex ratio [4,5]. Modify the above logistic model and the result is:

$$\frac{dN}{dt} = r \cdot N(t) \cdot \left(1 - \frac{N(t)}{K} - \gamma \cdot |M - 0.5| + \beta \cdot R(t)\right) \tag{4}$$

Since lamprey resources also have the ability to grow by themselves, we used S to represent the fixed growth rate of available resources for lampreys. C represents the resource consumption capacity of the lamprey population, multiplied by NR/K, and represents the consumption rate of available resources under the current lamprey population [6]. Therefore, the entire formula can express that the available resources in the ecosystem are still recovering while being eaten and consumed by the population. Therefore, the change of available resources R can be expressed as:

$$\frac{dR}{dt} = S - C \cdot \frac{N(t)}{K} \cdot R(t) \tag{5}$$

So far, the basic model has been established. By reviewing relevant literature, the following set of reasonable parameter values are obtained [3]:

$$r = 0.06;\ K = 500;\ \gamma = 1.2;\ \alpha = 0.06;\ \beta = 0.4;\ S = 0.1;\ C = 0.25;\ R_{th} = 0.5$$

We discuss the two situations of resource scarcity and resource abundance respectively. We set $R = 0.2$ to represent the initial resource scarcity situation, and $R = 0.8$ to represent the initial resource abundance. We solve the two situations through finite difference loops.

2.2. Multi - species competition model

The 2.1 model described above was analyzed for individual species of lamprey and for the entire ecosystem in which they are found. Here we focus on the multi-species relationships between lamprey and prey and predators.

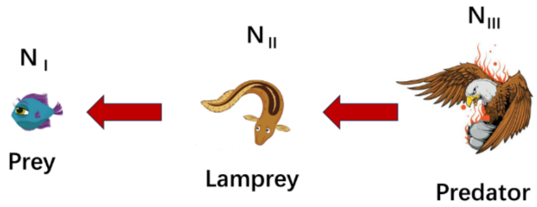

Figure 1. A food chain containing lampreys

We expanded the basic *Lotka-Volterra predator equation* (*L-V equation*) to implement a simplified food chain model between three species, and added gender correction to the growth rate in the *L-V equation*. N_I is used to represent the population of lamprey prey, and N_{II} is used to represent the population of lamprey predators. Based on the logistic model and the *L-V equation*, the changes in prey and predator populations can be expressed as [7,8]:

$$\frac{dN_I}{dt} = r_I \cdot N_I \cdot \left(1 - \frac{N_I}{K_I}\right) - C_I \cdot N_{II} \cdot N_I \tag{6}$$

$$\frac{dN_{III}}{dt} = r_{III} \cdot N_{III} \cdot \left(1 - \frac{N_{III}}{K_{III}}\right) - p' \cdot C_{II} \cdot N_{II} \cdot N_{III} \tag{7}$$

Among them, C_I represents the predation rate of lampreys on prey, and C_{II} represents the predation rate of predators on lampreys. The first term on the right side of the equation gives the changing trend of various groups themselves in the natural environment, while the second term describes the change caused by prey being killed by predators.

For lampreys, sex regulation is described by γ. Due to the existence of the food chain, the number of lampreys is reduced due to predation, and the number of lampreys is increased by preying on prey. Generally, predators always need to eat several meals before they are full and have the opportunity to raise a newborn baby, creating a conversion relationship between prey captured and newborn predators, with the conversion coefficient p, then the change in lamprey population can be expressed as:

$$\frac{dN_{II}}{dt} = r_{II} N_{II} \left(1 - \frac{N_{II}}{K_{II}} - \gamma M(1 - M)\right) - C_{II} N_{II} N_{III} + p \cdot C_I N_I N_{II} \tag{8}$$

Among them, C_{II} represents the predation rate of lampreys by predators, and p is used to describe the quantitative conversion relationship between prey and lampreys.

In this model, the available resource R for lampreys comes from the difference between the preys. Which is also a threshold. When $R < R_{th}$, the number of resource-rich males dominates. When $R > R_{th}$, the number of resource-poor males decreases. The change in the number of male lampreys can be expressed as:

$$\frac{dM}{dt} = -\alpha_{II} \cdot \left(\frac{1}{\beta_I} \cdot \frac{N_I}{K_I} - \frac{1}{\beta_{III}} \cdot \frac{N_{III}}{K_{III}} - R_{th}\right) \tag{9}$$

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3. Results

3.1. Impact of the sex ratio on the larger ecological system

Without gender adjustment, the male-to-female ratio in the lamprey population can be regarded as a constant. When predicting population size, the male ratio M is fixed, with 0.5 representing a 1:1 birth sex ratio, indicating gender balance. Thus, the correction for mortality due to sex competition is removed, leaving only the resource availability impact, returning the model to the basic logistics model.

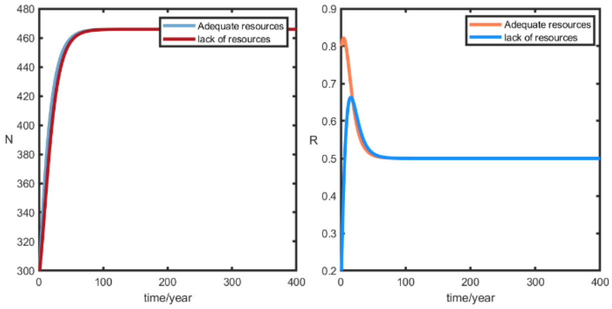

Figure 2. With both resource abundance and shortage, the lamprey population rapidly increases to a peak before gradually decreasing and stabilizing, consistent with the basic logistic model.

With gender adjustment, the male-female ratio adapts to the environment over time (Figure 3).

According to the model regulations, we regard R as resource availability as the relationship factor between the lamprey population and the larger ecosystem, reflecting the availability efficiency of the lamprey population in this ecosystem.

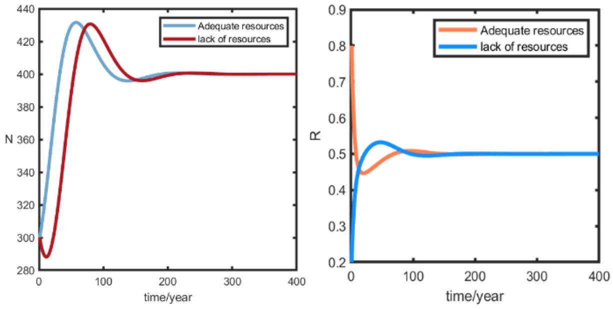

Figure 3. (Left) When resources are abundant, lamprey populations rapidly increase to a maximum before stabilizing; with scarce resources, the population decreases sharply, then slowly increases and stabilizes. In both cases, the final stable population is equal. **(Right)** With abundant initial resources, the population grows rapidly, leading to overconsumption and a drop in resource availability below 0.5. When initial resources are scarce, population and consumption decrease initially, then increase, causing the curve's inflection point to occur later than the initial resource abundance. This is consistent with population changes in the natural environment.

While this may benefit the lamprey population in the short term, this can degrade the environment for other species due to the conductivity and delay of the food chain. (This issue will be discussed further in the second model). Overall, gender-regulated lampreys contribute more to the sustainability of the larger ecosystem in which they reside.

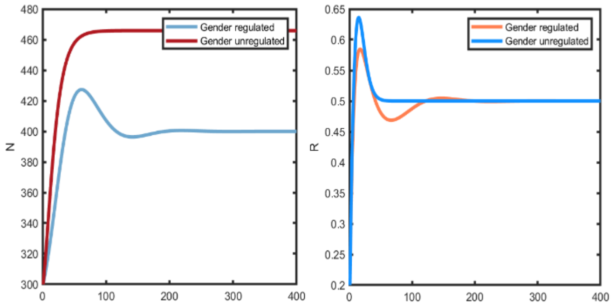

Figure 4. (Left) With gender adjustment, the population first peaks, then decreases, and stabilizes; without it, the population gradually increases to a higher stable value. This indicates that gender adjustment stabilizes the lamprey population within the maximum environmental carrying capacity, while the lack of adjustment leads to unsustainable growth. **(Right)** When the initial resources are low, both curves first rise and then fluctuate towards a stable value. With gender adjustment, our resource availability fluctuates less than without gender adjustment. This further confirms that populations under gender regulation are more sustainable.

3.2. Impact of the sex ratio on the stability of the ecosystem

Conduct stability analysis on the model ecosystem. In the *L-V equation*, it is easy to find the equilibrium point of the system, or fixed point. Solve for the number of preys N_I, lampreys N_{II} and predators N_{III} in the two cases of gender adjustment and no gender adjustment respectively (Figure 4).

Using the allowable error inequality:

$$|N(t) - N(\infty)| \leq \Delta \cdot N(\infty) \tag{12}$$

Calculate the stable time t and stable value N of the population in Figure 4, with and without gender adjustment, and determine the average population N from the initial time to t (Table 1). The stable value in the natural state can be obtained from the literature.

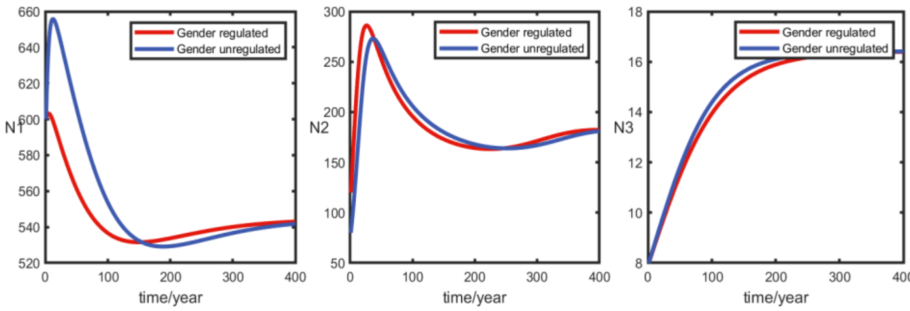

Figure 5. The number of preys, lampreys and predators will eventually tend to the same stable value N at a certain time t in both cases with and without gender adjustment.

Table 1. From the table we get that the food chain quantity matrix is $A = [N_I, N_{II}, N_{III}]$, the food chain quantity matrix is $A' = \left[N_I', N_{II}', N_{III}'\right]$ with gender adjustment, the quantity matrix of the food chain in the natural state is $A_0 = [N_{I,0}, N_{II,0}, N_{III,0}]$

		Prey	Predator	Lamprey
Stabilizing moment	Notation	t_1	t_2	t_3
	Value/year	260	240	250
Stable numbers	Notation	$N_{I,\infty}$	$N_{II,\infty}$	$N_{III,\infty}$
	Value/tail	543	180	20

Table 1. (continued)

Mean value	Gender unregulated	Notation	N_I	N_{II}	N_{III}
		Value/year	561.20864	209.67320	14.09250
	Gender regulated	Notation	$N_I{}'$	$N_{II}{}'$	$N_{III}{}'$
		Value/year	538.61383	210.96716	13.80574
Under natural conditions		Notation	$N_{I,0}$	$N_{II,0}$	$N_{III,0}$
		Value/year	543.67395	221.36748	10.26478

The degree to which the number of food chains deviates from the natural state in the two situations is quantified through Inverse Distance Weighting:

$$f = \frac{1}{d(A, A_0)} \tag{13}$$

Among them, d represents the Euclidean distance between the two matrices.

After calculation, the following results can be obtained:

$$d = 21.42130 > d' = 12.09587$$

Obviously, with gender adjustment, the number of preys, lampreys and predators are closer to the natural state than without gender adjustment. Thus:

$$f = 0.04668, f' = 0.08267$$

As expected, gender regulation promotes ecosystem stability, bringing each trophic level closer to natural values. Gender regulation is a manifestation of the lamprey's adaptation to the environment that is conducive to survival.

4. Discussion

The model in this article is reasonably improved based on the logistic growth model and combined with the *L-V equation*, enhances stability analysis and explains the relationship between lamprey sexual regulation and environmental resources. It is versatile and scalable, with modifiable differential equations to account for additional factors, The model uses reasonable descriptive metrics to account for species survival characteristics.

Due to insufficient data, Our model's accuracy is limited to the years covered by the data collected. We did not consider specific environmental factors either, such as water quality, pH value, sunlight, human intervention, etc. To improve it for studying lamprey biology, the differential equation can be modified for growth rate fluctuations under various scenarios. Analyzing ecosystems in different locations and obtaining more data would improve the accuracy of simulating lamprey sex regulation.

5. Conclusions

The results here reported show that the lamprey's gender adjustment has a positive effect on the larger ecosystem and the stability of the host ecosystem. The species can regulate its own sex ratio to maximize the use of resources in its environment. The ecosystem will move into a relatively stable pattern of development with gender adjustment and rationalizing resources. And sustainable development will offer the prospect of a more dynamic Lamprey population, rather than simply an increase in population size. In the face of broader ecological stability, this regulation enables nature's nutrient ratios to more closely resemble a cyclical natural state, thus making the entire food chain more stable and the entire ecosystem more stable.

The derivation of this model is based on purely mathematical implications, and the process can be completely reapplied to other species with variable sex ratios. This means that all species with adjustable sex ratios based on this rule have such positive implications. In fact, this class of organisms is also widespread in nature.

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