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Measurement of Moment of Inertia of Objects Based on the Law of Conservation of Angular Momentum

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Abstract: The rapid advancement of science and technology has rendered the moment of inertia a pivotal parameter in engineering, with significant applications in mechanical engineering and other domains. In undergraduate physics experiments, the determination of moment of inertia is predominantly confined to conventional techniques such as the three-wire pendulum method and the torsional pendulum method. However, in everyday scenarios, the objects requiring measurement are frequently irregular in shape. This paper, grounded in the principle of conservation of angular momentum, derives the correlation between the moment of inertia of the object in question and the angular velocity of a large disk. It translates the measurement of the object's moment of inertia into a measurement of the large disk's angular velocity. This approach not only enhances the precision of measurement but also simplifies and accurate the process of determining the moment of inertia of the object.

Keywords: Moment of inertia, Moment of inertia measurement, Conservation of angular momentum, Linear fitting

1. Introduction

Moment of inertia quantifies a rigid body's resistance to rotational acceleration about an axis. The determination of moment of inertia is a crucial aspect of university physics experiments [1-5] and holds profound significance in disciplines such as mechanical engineering, medicine, and biological engineering. Currently, the predominant laboratory techniques for assessing moment of inertia include the torsion pendulum method [6], the tower wheel method, and the three-wire pendulum method [7]. These methodologies are typically confined to rigid bodies with regular shapes, uniform mass distribution, and continuous density. For objects of irregular form, measuring their moment of inertia often necessitates intricate integral computations [8]. This paper introduces a novel instrument for measuring moment of inertia, which leverages the law of conservation of angular momentum to address the limitations of conventional techniques. This apparatus streamlines the measurement process and enhances accuracy, allowing for the straightforward determination of the moment of inertia of irregularly shaped objects.

2. Instrument And Methods

2.1. Laboratory equipment

Moment of inertia measuring apparatus, depicted in Figure 1, primarily consists of a large disk, a motor, a small flywheel, a photoelectric gate, a counter, and a laser positioner. The small flywheel and the motor are symmetrically arranged.

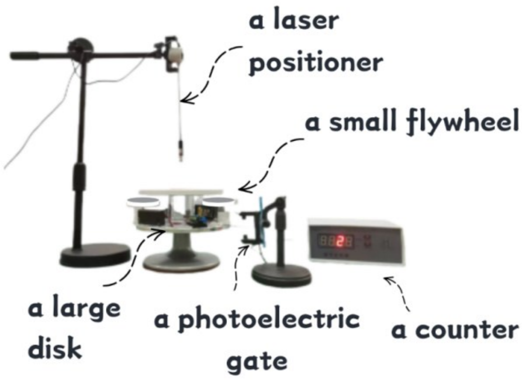

Figure 1: Actual image of the new moment of inertia measuring instrument

The small flywheel (1) has a diameter of 8 cm and a thickness of 1 cm. It features a central shaft with a diameter of 2 mm, which is securely fixed to the shaft of the DC motor (3). By adjusting the governor of the DC motor, the angular velocity of the flywheel can be altered [9]. The electric motor is powered by a 9-volt battery. The symmetric placement of the small flywheel serves to maintain the level of the large disc, reduce the frictional torque on the rotating shaft, and extend the measurement range.

The large disc is mounted on the base via a rotating shaft, with the base secured in place, allowing the large disc to rotate around its axis. As depicted in Figure 2, the large disc has a diameter of 23 cm, and the rotating shaft incorporates a high-speed, silent bearing (2) that exhibits excellent smoothness and quiet operation. To minimize the impact of friction, lubricating oil is also applied to the rotating shaft. When the small flywheel is in motion, the minimal friction torque beneath the large disc's rotating shaft allows the system comprising the large disc and the small flywheel to be treated as approximately conserving angular momentum. In accordance with the law of conservation of angular momentum, as the small flywheel rotates, the large disc will rotate in the opposite direction.

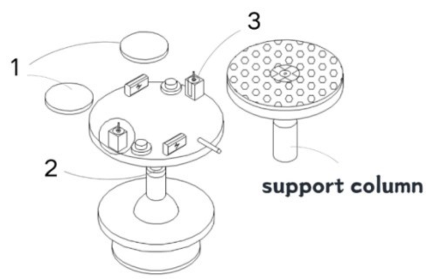

Figure 2: Schematic diagram of the large disc structure

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The counter and photogate are utilized to measure the number of rotations (n) and the time interval (t) of the large disk, respectively. Consequently, the period (T) can be calculated as $T = t/n$, and the angular velocity (ω) of the large disk can be determined as $\omega = \frac{2\pi}{T} = \frac{2\pi n}{t}$.

The laser locator is designed to ensure that the object to be measured is positioned along the central axis of the large disc. As illustrated in Figure 3, the crosshair of the laser should be aligned with the geometric center of the large disc. Subsequently, the object to be measured is placed on the large disc, with care taken to ensure that the center of the object aligns with the crosshair of the laser.

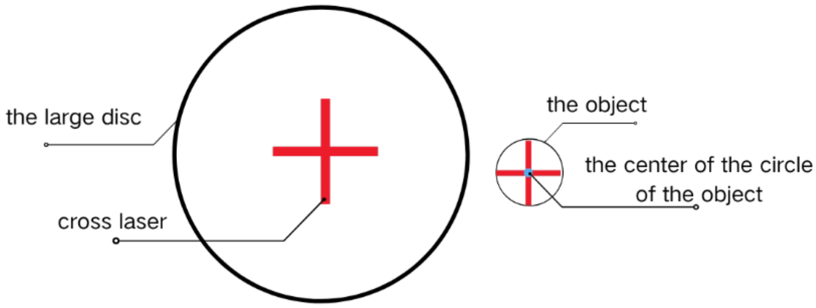

Figure 3: Schematic representation of the alignment process for the crosshair laser.

2.2. Conservation of Angular Momentum in the “Large Disc + Small Flywheel” System

The system, comprising a large disc and a small flywheel, is depicted in Figure 4. Upon initiating the motor, the two small flywheels rotate at angular velocities ω_{11} and ω_{12}, respectively, while the large disc rotates in the opposite direction at an angular velocity ω_0, thereby maintaining the angular momentum of the system at zero. This can be expressed as:

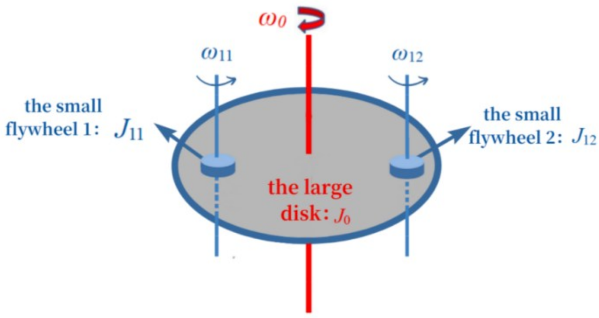

Figure 4: Large disc + small flywheel

$$J_{11}\omega_{11}+J_{12}\omega_{12}=J_0\omega_0 \tag{1}$$

In equation (1), J_{11} and J_{12} correspond to the moments of inertia of the small flywheels, respectively, while J_0 represents the moment of inertia of the measuring apparatus.

2.3. Conservation of Angular Momentum in the “Large Disk + Body + Small Flywheel” System

If an object is placed at the center of the large disk while maintaining a constant rotational speed of the small flywheel, the time taken for the large disk to complete N revolutions can be measured using

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a photogate and a counter. Based on this data, the angular velocity ω_0' of the large disk after placing the object can be calculated.

As shown in Figure 5, after the object is placed, the angular momentum of the system composed of the large disk, the object, and the small flywheel remains conserved.

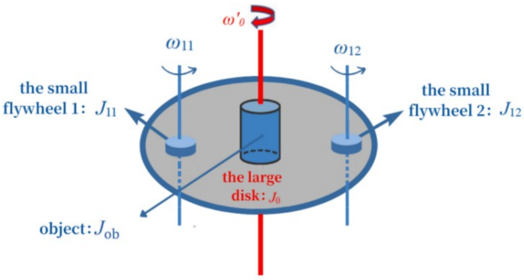

Figure 5: Large disk + object + small flywheel

Given that the angular velocity of the small flywheel is significantly greater than that of the large disk, the angular velocity of the small flywheel before and after placing the object remains approximately the same. Consequently, the following is calculated as:

$$J_{11}\omega_{11}+J_{12}\omega_{12}=\left(J_0+J_{ob}\right)\omega_0' \tag{2}$$

where J_{ob} represents the moment of inertia of the object.

2.4. The relationship between J_0 and J_x

By combining equation (1) with equation (2), a new equation is derived(3):

$$J_0\omega_0=\left(J_0+J_{ob}\right)\omega_0' \tag{3}$$

And then formula (4) can be derived from formula (3) as follows:

$$J_0=\frac{\omega_0'}{\omega_0-\omega_0'}J_{ob} \tag{4}$$

After that, we replace the previously discussed object with a uniform cylinder having a radius of R and a mass of m, as illustrated in Figure 6.

$$J_{cy}=\int r^2 \mathrm{d}m=\frac{1}{2}mR^2 \quad (5)$$

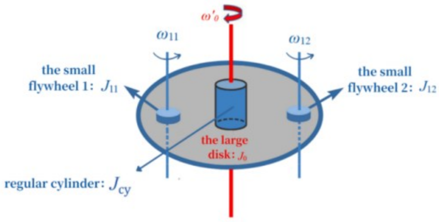

Figure 6: Large disc + cylinder + small flywheel

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Using formula (5), which calculates the moment of inertia for the uniform cylinder, formula (4) can be restructured to yield formula (6).

$$J_0=\frac{\omega_0^{'}}{\omega_0-\omega_0^{'}}J_{cy} \tag{6}$$

Utilizing the theoretical value of the moment of inertia for a uniform cylinder, we determined the moment of inertia J_0 of the large disk as follows:

$$J_0 = 1.878 \times 10^{-3} \left(\mathrm{kg} \cdot \mathrm{m}^2\right) \tag{7}$$

By reordering the terms in Equation (3) and substituting variable J_{ob} with J_x, the computational formula is derived for the moment of inertia of the object under consideration:

$$J_x=\frac{\omega_0-\omega_0^{'}}{\omega_0^{'}}J_0 \tag{8}$$

Therefore, by measuring the angular velocities ω_0 and $\omega_0^{'}$ of the large disk prior to and subsequent to the placement of the object to be measured, the value of J_x can be determined.

3. Results

3.1. The Value of J_0

This paper utilizes four standard objects with regular shapes. The mass and radius of each object are measured multiple times, and the computed values of J_0 are averaged to mitigate random errors, as presented in Table 1 and Table 2.

Table 1: The theoretical value of moment of inertia of the standard object

Standard object	m/(g)	r/(cm)	J_{th}/(10^{-4}kg · m^2)
1	278.81	4.48	2.798
2	218.17	3.97	1.719
3	315.24	4.01	2.535
4	482.52	4.91	5.816

Table 2: The measured data of moment of inertia of the large disc

Standard object	ω_0/ (rad/s)	$\omega_0^{'}$/ (rad/s)	J_0/(10^{-4}kg · m^2)
1	2.933	1.178	1.878
2	3.687	1.924	1.876
3	3.598	1.531	1.877
4	4.552	1.111	1.878

The moment of inertia, J_0 , of the large disk is as follows: J_0=1.878×10^{-3}(kg · m^2) (7).

3.2. Correction of measuring formula of moment of inertia

To ascertain the measurement accuracy of the developed measuring device, this study takes into account potential confounding factors such as friction that may influence the measurement outcomes. Ten regularly shaped cylinders with a uniform mass distribution are selected as the measurement subjects, with their theoretical values serving as the standard reference points. The mass, radius, and

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moment of inertia with respect to the central axis for each of these subjects are measured on six separate occasions to minimize random errors, and the mean of these measurements is computed. The resulting data are presented in Tables 3 and 4. Additionally, the data contained in Table 4 are subjected to linear fitting analysis [11].

Table 3: The theoretical value of moment of inertia of the object to be measured

Object to be measured	m/(g)	r/(cm)	J_{th}/(10^{-4}kg · m^2)
1	190.98	4.48	1.917
2	256.05	3.97	2.017
3	301.08	4.01	2.421
4	335.81	4.91	4.048
5	352.88	4.46	3.510
6	366.05	3.11	1.770
7	371.91	3.86	2.771
8	405.14	5.12	5.310
9	411.52	5.37	5.933
10	435.17	5.52	6.630

Table 4: The measured value of moment of inertia of the object to be measured

Object to be measured	ω_0/ (rad/s)	$\omega_0^{'}$/ (rad/s)	J_{me}/(10^{-4}kg · m^2)	relative error
1	1.792	1.623	1.956	2.04%
2	2.786	2.522	1.966	2.59%
3	1.902	1.679	2.494	3.03%
4	2.012	1.649	4.133	2.12%
5	1.949	1.633	3.633	3.53%
6	2.453	2.247	1.721	2.75%
7	2.307	2.019	2.679	3.32%
8	2.111	1.652	5.217	1.75%
9	2.543	1.941	5.823	1.85%
10	2.698	1.985	6.745	1.73%

The theoretical moment of inertia, J_{th}, for the object under measurement is adopted as the standard reference value. This value is compared with the measured moment of inertia, J_{me}, obtained using the instrument. The measurement results are then corrected. In Figure 7, the measured values (J_{me}) are plotted on the horizontal axis, while the theoretical values (J_{th}) are plotted on the vertical axis. The correction outcomes are presented in Table 5.

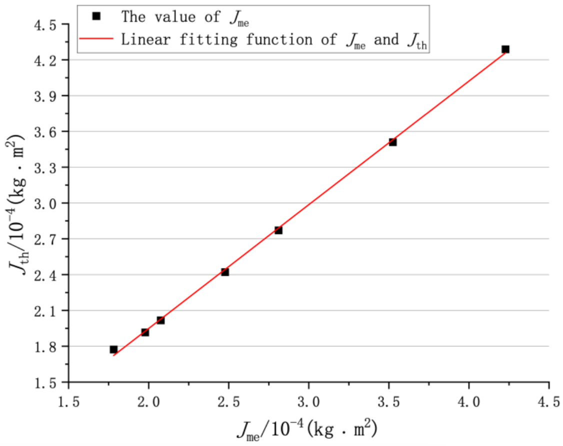

Figure 7: Diagram of the relationship between J_{me} and J_{th}.

The following modified formula has been derived:

$$J_x=1.96918\times10^{-3}\times\frac{\omega_0-\omega_0^{'}}{\omega_0^{'}}-1.2429\times10^{-5} \quad (9)$$

Table 5: The correction of moment of inertia of the object to be measured

Object to be measured	ω_0/ (rad/s)	$\omega_0^{'}$/ (rad/s)	$J_{me}/(10^{-4}$kg · m$^2)$	relative error
1	1.561	1.415	1.908	0.47%
2	2.566	2.312	2.039	1.05%
3	2.325	2.057	2.441	0.85%
4	1.765	1.458	4.022	0.64%
5	2.143	1.810	3.499	0.32%
6	1.875	1.712	1.751	1.11%
7	1.986	1.733	2.751	0.73%
8	2.441	1.912	5.324	0.26%
9	2.758	2.112	5.899	0.58%
10	2.352	1.753	6.604	0.38%

3.3. Comparative analysis of measurement methods

To facilitate comparison with the measurement outcomes of conventional experimental techniques, we selected two arbitrary objects from a set of ten and determined their moment of inertia, J_{twp}, relative to the central axis utilizing the three-wire pendulum method [12]. With the theoretical value, J_{th}, serving as the reference standard, the relative error was computed. The corresponding results are presented in Table 6.

Table 6: Comparison of results of measurement methods

Object to be measured	J_{th}/ $(10^{-4}$kg · m$^2)$	J_{me}/ $(10^{-4}$kg · m$^2)$	relative error of J_{me}	J_{twp}/ $(10^{-4}$kg · m$^2)$	relative error of J_{twp}
1	1.921	1.910	0.52%	1.816	5.21%
2	4.289	4.260	0.69%	4.461	3.96%

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Table 6 illustrates the comparative average relative errors for the measurement of objects 1 and 2. The instrument developed in this study yielded average relative errors of 0.52% for object 1 and 0.69% for object 2. In contrast, the three-wire pendulum method resulted in average relative errors of 5.21% for object 1 and 3.96% for object 2. The relative errors associated with the measurements obtained from the instrument proposed herein are consistently lower than those obtained using the three-wire pendulum method [13-15].

Analysis of the measurement data indicates the presence of inherent measurement errors within the instrument. These errors are potentially attributed to minor displacement deviations in the initial placement position of the object, as well as to slight shifts of the object under the influence of centrifugal force during the rotation of the large disk. These discrepancies contribute to the observed measurement inaccuracies.

4. Conclusion

This paper presents a novel moment of inertia measuring instrument, which operates on the principle of the conservation of angular momentum. This device effectively and accurately measures the moment of inertia of an object by transposing the measurement into the determination of the angular velocity of a large disk, thereby simplifying the measurement process and enhancing ease of operation.

Additionally, the instrument benefits from a straightforward calculation formula, with its measurement accuracy further enhanced through subsequent refinements. The device not only facilitates students' comprehension and application of the conservation of angular momentum and moment of inertia but also offers practical assistance for industrial measurement applications. In terms of functionality, the measuring instrument is particularly well-suited for use in university physics experiments, where precision and educational value are paramount [16].

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