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Active control of membrane structure vibration based on sliding mode control

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Abstract. This paper studies the active vibration control of membrane structure by piezoelectric actuator. Firstly, the nonlinear vibration equation is established for membrane structure and the vibration equation is supplemented on the basis of the system equation established by Liu Xiang et al., and then the multi-mode dynamic model is obtained by Galerkin decomposition for the four-degree-of-freedom modes. After that, the Sliding Mode Controller is designed to suppress the large initial displacement vibration of the membrane structure and the continuous external excitation of the membrane structure. The stability of the controller is proved by using the Lyaplov stability theory and the effectiveness of the controller is verified by numerical simulation. In addition, the control effect of synovial control is compared with that of negaive velocity feedback control, which proves that synovial control has better robustness.

Keywords: Menbrane structure,Active control,SMC controller,Nonlinear vibration.

1. Introduction

At present, different kinds of membrane materials have been widely used in the aerospace field, because of its light weight, easy to fold, large deformation, high flexibility, low damping coefficient of the advantages, but also has the same defects, such as its low damping coefficient, high flexibility, small oscillations in the space will gradually become bigger, a certain loss of performance, and harm the membrane itself structure, and even bring serious accidents. Therefore, how to control large-amplitude vibration is currently a subject of great concern.

The active control strategy has carried out in-depth theoretical and practical research. Many scholars use mathematical means for active control. For example, an actuator is established so that the film can generate mechanical motion according to the control signal.Lu.YF et al established the control equations based on the geometric nonlinear theory of the plate, considering the modal control force induced by the actuator.A polyvinylidene fluoride (PVDF) actuator is used to realize the active vibration control of pre-tensioned Kapton film, and the optimal criterion for the actuator arrangement and energy distribution are also proposed to realize vibration suppression [1, 2]. Yang Liu proposed an actuator and sensor arrangement method based on input-output matrix singular value decomposition (SVD) to ensure the modal controllability and observability of the system [3].

In addition, a number of scholars have adopted the establishment of mechanical models to design controllers, such as Liu Xiang et al studied the position optimization of piezoelectric actuator and active vibration control of membrane structure, and verified the control performance of the optimal position

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[4]. Later, he used the deformation assumption and the nonlinear finite element method to establish the dynamic equations and nonlinear dynamics model of the membrane structure, and designed an adaptive controller to suppress the large-amplitude vibration of the membrane structure and verified its stability [5-8]. IChen, Tian-Ze et al developed a dynamical model of large-amplitude vibration for accurate dynamic control of a solar sail.An attitude tracking controller and a nonlinear vibration controller based on the cable were also designed to control the attitude motion and the flexible vibration of the solar sail [9]. Inman et al explored the piezoelectric actuator for the membrane mirror in literature for the vibration, and evaluated various control strategies in their study and designed an LQR closed-loop controller to verify its feasibility [10-12]. Toshiki Hiruta et al proposed a lightweight flexible membrane structure vibration suppression technique based on dielectric elastomer actuator (DEA), designed a controller based on a recognized membrane model, and conducted vibration control experiments [13].

2. Nonlinear vibration equation for thin membrane structure

The studies in this paper are all based on rectangular thin film structures of thickness h, length and width m and n respectively and with clamped boundaries. Where M(0,0) is the center point of the film, A(-0.5m,-0.5n), B(-0.5m,0.5n), C(0.5m,0.5n), D(0.5m, 0.5n) are the four vertices of the rectangle of the film. The piezoelectric actuator can realize precise and fast motion control and nonlinear vibration control by applying control voltage to generate bending moment, which has the advantages of compactness and lightweight, high bandwidth and high accuracy, and the vibration control effect can be improved by changing the number of actuators. Film specific parameters can be obtained from [5].

Assuming uniform material mass distribution and neglecting the in-plane displacement of the film, the structural equation of the film can be obtained from [1] as:

$$\rho h\frac{\partial^2 w}{\partial t^2} - \left(N_{x0}\frac{\partial^2 w}{\partial x^2} + \mathrm{N}_{y0}\frac{\partial^2 w}{\partial y^2} + 2\mathrm{N}_{xy0}\frac{\partial^2 w}{\partial x\,\partial y}\right) - \frac{Eh}{2(1-\mu^2)}\left[\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^2 w}{\partial x^2} + \mu\left(\frac{\partial w}{\partial y}\right)^2\frac{\partial^2 w}{\partial y^2}\right] - \frac{Eh}{2(1-\mu^2)}\left[\mu\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^2 w}{\partial y^2} + \left(\frac{\partial w}{\partial y}\right)^2\frac{\partial^2 w}{\partial x^2}\right] - \frac{Eh\mu}{1+\mu}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x\,\partial y} = \mathrm{p(x,y,t)}. \tag{1}$$

In the above equation, ρ is the bulk density of the membrane structure; h is the thickness; w indicates displacement in the z (lateral) direction; N_{x0} and N_{y0} are in-plane principal forces per unit length in x and y directions; N_{xy0} is the in-plane shear force per unit length; E is Young's modulus; μ is the Poisson ratio; $p(x,y,t)$ is the external load.

From [5], in order to obtain a set of modal equations, the decomposition into spatio-temporal functions is done by using Galerkin decomposition. The following equation can be obtained:

$$\mathrm{w(x,y,t)} = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\phi_{ij}(x,y)X_{ij}(t). \tag{2}$$

$$\phi_{ij}(x,y) = \sin\left(\frac{i\pi x}{m}\right)\sin\left(\frac{j\pi y}{n}\right). \tag{3}$$

In the above equation, $X_{ij}(t)$ is modal coordinate, $\phi_{ij}(x,y)$ is mode shape functions. This paper considers the first four orders of modes, $\boldsymbol{i}$ and $\boldsymbol{j}$ are taken as 1,2,3,4.

From [1], the system control equation is given as follows:

$$\ddot{X}_{ij} + 2\zeta_0\omega_{ij}\dot{X} + K_1X_{ij} + K_3X^3 = P + \Phi. \tag{4}$$

In the above equation, $\boldsymbol{\zeta_0}$ is structural intrinsic modal damping ratio, $\boldsymbol{\omega_{ij}}$ is nonlinear frequency, K_1 is linear component of membrane frequency, K_3 is nonlinear component of membrane frequency; Φ is the control force provided by piezoelectric actuators, P is external loads on the surface.

And by [5] K_3X^3 can be written as $\sum_{i,j=1}\sum_{u,v=1}\sum_{t,s=1} a_{ijuvts}\,X_{ij}\,X_{uv}\,X_{ts}$

$$\ddot{X}_{ij} + 2\zeta_0\omega_{ij}\dot{X} + K_I X_{ij} + \sum_{i,j=1}\sum_{u,v=1}\sum_{t,s=1} a_{ijuvts}X_{ij}X_{uv}X_{ts} = P + \Phi. \tag{5}$$

In this paper, we consider the first four orders of vibration modes ,According to equation (2) and equation(3), the fourth-order nonlinear coupled partial model is obtained from [5].

3. Control effect and comparison between sliding Mode Control and Negative Velocity Feedback Control

3.1. Sliding Mode Controller

The model in this paper refers to Sliding Mode Control (SMC). Sliding Mode Control (SMC), also known as variable structure control, is a nonlinear control method that has attracted wide attention in the field of control theory in recent years. The core idea lies in the design of a switching hyperplane, so that the state of the system can be purposefully changed according to the current state. In terms of applications, sliding mode control has been widely used in various fields. For example, in the aerospace field, sliding mode control can be used for the tasks of attitude control and trajectory tracking of aircraft. Nsubuga Latifu investigated a full-order sliding film control method for distributed finite-time attitude synchronization tracking of multiple spacecraft. transparency, pseudo-sliding mode control is proposed as a viable option for designing complex six-degree-of-freedom aircraft automatic flight control systems; in the field of robot control, sliding mode control can be used for robot path planning, motion control, etc., for example, SEUNGJIN KOH establishes a nonlinear model of two-wheeled counterbalanced mobile robot, and utilizes the sliding mode control to improve the localization and anti-jamming control of the two-wheeled counterbalanced mobile robot. control.

In this paper the controller s is given by [14],Deform equation(5) to the following form:

$$\ddot{X}_{ij} = -\left(2\zeta_0\omega_{ij}\dot{X} + K_I X_{ij} + \sum_{i,j=1}\sum_{u,v=1}\sum_{t,s=1} a_{ijuvts}X_{ij}X_{uv}X_{ts}\right) + P + \Phi^{ij} \tag{6}$$

$$\dot{s} = \left(\ddot{X}_d + \left(2\zeta_0\omega_{ij}\dot{X} + K_I X_{ij} + \sum_{i,j=1}\sum_{u,v=1}\sum_{t,s=1} a_{ijuvts}X_{ij}X_{uv}X_{ts}\right) - P - \Phi\right) + \kappa(X_d - X). \tag{7}$$

There is always a control voltage Φ can satisfy $\dot{s} = -sgn(s)\xi - ks < 0$, to fulfill $s\dot{s} < 0$. So the stability of the sliding mode controller is proven。

The output force of piezoelectric actuator Φ is:

$$\Phi = \ddot{X}_d + \kappa(\dot{X}_d - \dot{X}) + 2\xi_0\omega_{ij}\dot{X} + K_I X_{ij} + \sum_{i,j=1}\sum_{u,v=1}\sum_{t,s=1} a_{ijuvts}X_{ij}X_{uv}X_{ts} - P + \mathrm{sgn}(s)\,\zeta + ks. \tag{8}$$

In the above equation,k and κ is sliding mold surface parameters , $sgn(s)$is a step function, $\zeta$is the thickness of the slide surface.

3.2. Negative velocity feedback control

From [1], it is known that real-time vibration control can be realized by negative velocity feedback based algorithm, and the control voltage and modal control force are defined as:

$$U = -G\dot{X}(t). \tag{9}$$

$$\Phi_{ij}^{c}(t) = -A_{ij}G\dot{X}(t). \tag{10}$$

$$A_{ij} = \frac{4E_a\left(h + h_p\right)}{\rho hmn}\left[d_{31}\left(\frac{i\pi}{m}\right)^2 + d_{32}\left(\frac{j\pi}{n}\right)^2\right]\int_{y_1}^{y_2}\int_{x_1}^{x_2}\sin\left(\frac{i\pi x}{m}\right)\sin\left(\frac{j\pi y}{n}\right)dxdy. \tag{11}$$

Then the system control equation can be written as:

$$\ddot{X}_{ij} + 2\zeta_0\omega_{ij}\dot{X} + K_1X_{ij} + \sum_{i,j=1}\sum_{u,v=1}\sum_{t,s=1} a_{ijuvts}X_{ij}X_{uv}X_{ts} = P + \Phi_{ij}^c. \tag{12}$$

From equation (9), it follows that there is a clear linear positive correlation between control voltage $U$and control gain G.This means that as the voltage increases, the control effectiveness of the system increases accordingly. To ensure stable and safe operation of the actuator, this paper specifically limits the voltage range to ±1000V. In addition, the magnitude of A_{ij} is closely related to that of the actuator, and may have a negative value. It is important to note that when $\Phi_{ij}^c$is positive, the use of the current control method may significantly exacerbate the vibration, which in turn leads to instability of the system modes. Therefore, in order to maintain a stable control effect, we have to set up separate logic to keep Φ_{ij}^c within the negative range at all times. Under the condition of constant gain coefficient, the size of the modal driving factor is positively correlated with the control effect, i.e. the larger the factor, the more significant the control effect.

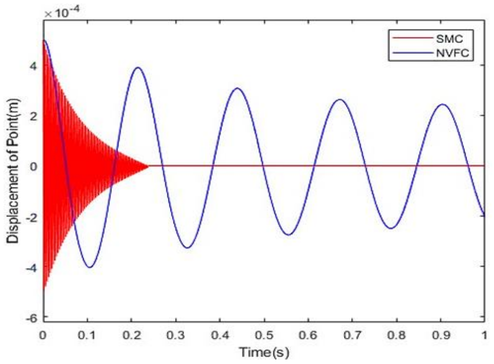

Figure 1. Comparison between SMC and NVFC

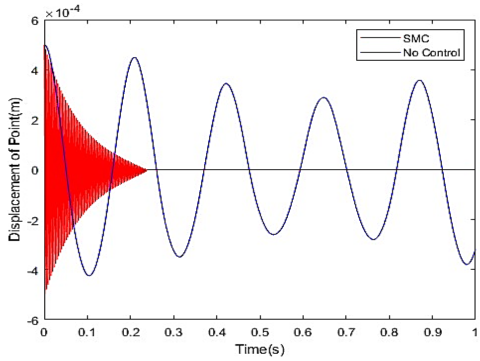

Figure 2. Comparison between SMC and No Control

3.3. *Comparison of the two control effects*

Displacement of point curve images are obtained by solving differential equation(5) and equation (12) in **Simulink**. Through detailed comparison and analysis of Figure 1 and Figure 2, this study finds that the sliding mode control (SMC) exhibits significant advantages over the traditional negative feedback control strategy in terms of control performance. Specifically, the sliding mode control, with its unique dynamic characteristics, achieves faster convergence speed and better control effect, thus significantly improving the overall performance of the control system.

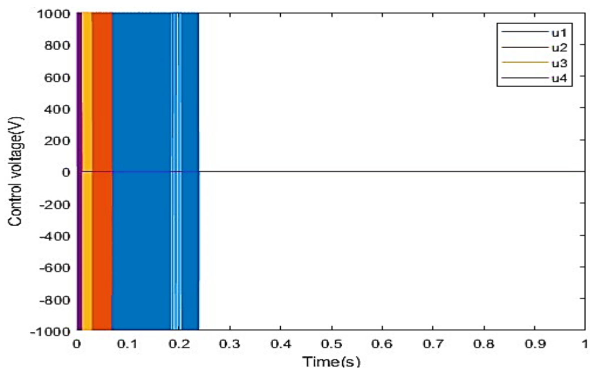

Figure 3. The control voltage of SMC

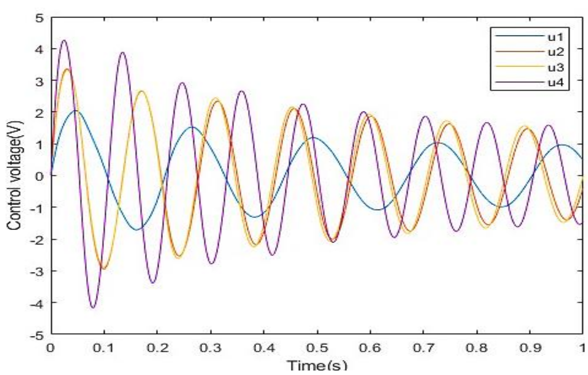

Figure 4. The control voltage of NVFC

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However, from **Figure 3** and **Figure 4**, it can be seen that there are still potential problems and non-negligible challenges in exploring the application of sliding mode control in depth. Among them, the SMC controller generates relatively high control voltage values during operation, and its voltage rate of change also shows a more rapid trend. This demand for high voltage and high voltage variability not only requires the control system to have a higher energy input capacity to maintain its stability, but also puts forward more stringent standards for the design parameters and performance requirements of the controller.

4. Research on the effect of nonliner external excitation vibration control

From [1]:

$$p_{ij}=f\delta(x-x_0)\delta(y-y_0)\sin\left(w_{ij}t\right). \tag{13}$$

As can be obtained from **Figure 5**, after comparing the displacement curves in the no-control state with those under the sliding mode control strategy, it can be clearly observed that the SMC control has a significant effect on improving the actuator performance. In the no-control state, the displacement curve of the actuator presents an irregular oscillatory waveform, and this instability may lead to the degradation of system performance or even failure. However, when SMC control is introduced, these irregular oscillatory waveforms are effectively converged to a sinusoidal waveform with clear periodicity, which significantly improves the stability and controllability of the system. And it can be obtained from **Figure 6** that the SMC control enables the actuator to respond quickly and reach the target position accurately by adjusting the control voltage in real time. However, this advantage depends on the control voltage being maintained at full power from time to time. While full power ensures that the actuator has sufficient drive to overcome various resistances and loads, maintaining high power for a long period of time can lead to a number of potential problems. When the membrane is continuously excited, the sliding mode controller can quickly converge the waveform to a smooth exciting curve.

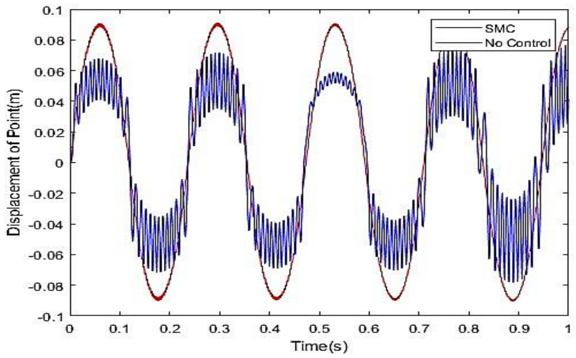

Figure 5. Comparison of SMC and No Control

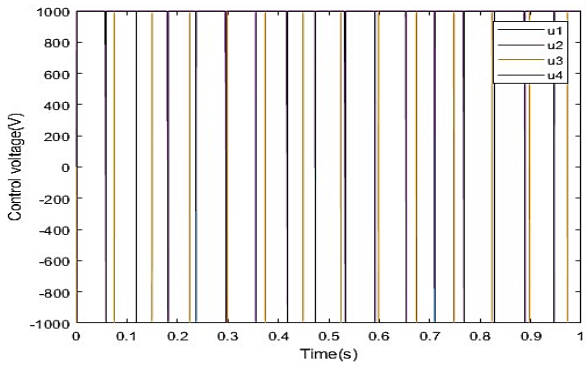

Figure 6. Control Voltage of SMC

5. Conclusion

In this paper, the membrane multi-degree-of-freedom free vibration control is simulated; the sliding film algorithm is adopted for the piezoelectric actuator, and the sliding film controller is designed to inhibit the large vibration of the membrane's initial displacement as well as the two cases that the membrane is continuously excited by the external excitation, and the displacement curves are compared with those of the no-control state; and the comparison of the piezoelectric actuator's control effect in the negative velocity feedback and the sliding film control is explored.The main conclusions obtained during the study can be summarized as below:(1) compared with negative velocity feedback control, sliding mode control has better control effect under similar control parameters. (2) When the membrane is continuously excited, the sliding mode controller can quickly converge the waveform to a smooth exciting curve.

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