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Pricing Chooser Option

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Abstract: With the rapid development of financial markets and the increasing diversification of investment instruments and choices, exotic derivatives like the Chooser option have emerged. This special type of option contract grants holders the right to choose whether the option is a put or a call at the selection date. This flexibility facilitates investors' hedging policies and diversification to spread risk and enhance returns. However, pricing it appropriately becomes a difficult task because of its complexity. This paper first aims to price the Chooser option in two ways: the N-period Binomial tree model and the Black-Scholes model combined with Monte Carlo simulations. Then, the results from both approaches are compared and it is found that the outcomes are approximately equivalent when the parameters are held constant. Building on this finding, the computational efficiency of both methods in Python is analyzed, with fixing N (period) at 100. The results indicate that the running time of the Black-Scholes model exceeds that of the N-period Binomial tree model.

Keywords: component, Chooser option, Binomial tree model, Black-Scholes model, Monte Carlo.

1. Introduction

1.1. Background information

As demand for corporate finance rises and personal wealth expands, more investors are entering the financial market, resulting in an unprecedented boom and growing uncertainty. To maximize benefits and minimize risks in the changing market, participants are constantly adjusting their asset portfolios and investment strategies by using different financial instruments. Investors first focused on investing in primary assets such as real estate and stocks. As a result of the globalization of the economy, leading to more cross-border transactions in the financial markets, derivatives emerged and were traded in 1973, which are used first to hedge or speculate on exchange rate risks and then hedge the risks of stock [1].

Based on the widespread use of standard options, exotic options have been derived to create more financial opportunities [2]. Take the chooser option as an example; it allows the holder to decide whether to call or put the option on any day prior to the exercise date. This flexibility is usefulness in hedging strategies cannot be matched by traditional options [3]. Nevertheless, there

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are also disadvantages associated with the chooser options. Typically, the price is relatively high due to the greater number of choices available [4]. Furthermore, It is difficult for investors to price it as non-standard strike conditions, multiple underlying assets or depending on certain specific events. This article aims to address this issue by concentrating on the pricing of Chooser options, utilizing both the N-period Binomial tree model and the Black-Scholes model in conjunction with Monte Carlo Simulation.

This paper will first introduce the concepts underlying the two pricing models. In the second part, details like the derivation of these models will be explored. The third section will display experiments in Python and then compare the results and operational speeds of the two models by substituting sample data. The conclusion is presented in the final part.

1.2. Literature review

The concept of the chooser option was first introduced by Mark Rubinstein [5]. The exploration of chooser options experienced significant advancements in 2009 when Detemple and Emmerilin offered a detailed analysis of American chooser options' exercise regions and pricing behaviors [6]. In 2012, Hanyang Financial Engineering Lab made a notable contribution by releasing a comprehensive financial engineering model for the chooser option. This includes pricing compound choice options and straddles, expanding the understanding and diversity of the choice options framework. The implementation of quantitative strategies for chooser options began to materialize in 2015, marked by the development of C++ test instances.

This paper not only provides the analytical solution of the option pricing model but also provides Python code for extensive application and verification to further explore the application of chooser options within financial engineering.

2. Methods

2.1. Pricing based on N-period Binomial tree model

The binomial tree model, one of the most frequently employed models for pricing [7], constitutes a foundational concept in finance and quantitative analysis. Its simplicity allows it to be effectively and widely applied to the pricing of complex financial derivatives [8]. The formula for pricing European options, as derived from the work of Fischer and Myron (1979), can be obtained through the application of the binomial tree model:

$$X_0 = \frac{1}{1+r}\left(\frac{1+r-d}{u-d}F\left(S_0u\right) + \frac{u-1-r}{u-d}F\left(S_0d\right)\right) = \frac{1}{1+r}E^Q\left[F\left(S_1\right)\right]\ [9]. \tag{1}$$

where q is the probability

$$q = \frac{1+r-d}{u-d}. \tag{2}$$

For the chooser option, assume the stock price has increased "i" times within the time range of 0 to T. "u" and "d" are the upward and downward price movements. The stock price can be calculated at time T:

$$S_T^i = S_0 \cdot u^i \cdot d^{T-i}. \tag{3}$$

The return of call and put options at T is:

$$C_T = \max\left[S_T - K, 0\right], P_T = \max\left[K - S_T, 0\right]. \tag{4}$$

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Substitute the above results into the formulae derived in section 1.3 yields the results that the price of the call option and put at choosing time U is

$$C_U^i = e^{-r\Delta t}\left(q \cdot C_{U+1}^{i+1} + (1-q)C_{U+1}^i\right), P_U^i = e^{-r\Delta t}\left(q \cdot P_{U+1}^{i+1} + (1-q)P_{U+1}^i\right). \quad (5)$$

Where q is the possibility of and Δt :

$$q = \frac{1+r-d}{u-d}, \Delta t = \frac{T}{N}. \quad (6)$$

According to the nature of the chooser option, one can decide to call or put at time U, the value of the chooser option can be shown as $V_U = \max(C_U, P_U)$. Therefore, the inverted rollout in the $U-1$ period option price, and so on to time 0:

$$V_{U-1}^i = e^{-r\Delta t}\left(q \cdot V_U^{i+1} + (1-q)V_U^i\right). \quad (7)$$

....

$$V_0^i = e^{-r\Delta t}\left(q \cdot V_1^{i+1} + (1-q)V_1^i\right). \quad (8)$$

Finally, summarise and group the above formulas into one general equation, n is integer $\in (1, U)$:

$$X_{U-n} = e^{-nr\Delta t}\sum_{j=0}^{n}\binom{n}{j} q^j (1-q)^{n-j} V_U^{i+j}. \quad (9)$$

2.2. Pricing based on Black-Scholes model

As N gradually approaches positive infinity, Black and Myron Scholes (1973) derived the BS model (Black-Scholes model) to be simpler and more efficient. The pricing of European options using the Black-Scholes model results in the following expression:

$$C_0^{BS} = \lim_{N\to\infty} X_0^{(N)} = S_0\Phi(d_1) - Ke^{-rT}\Phi\left(d_1 - \sigma\sqrt{T}\right). \quad (10)$$

where d_1 and Φ is

$$d_1 = \frac{\log\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}, \Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{y^2}{2}} dy. \quad (11)$$

The BS formula for a European call option is

$$C_0 = S_0 N(d_1) - Ke^{-rT}N(d_2). \quad (12)$$

In this equation, S_0 represents the stock price at time *0*, and *N* is the cumulative standard normal distribution function,

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}, d_2 = d_1 - \sigma\sqrt{T}. \quad (13)$$

K is the strike price. r is the risk-free interest rate. T is the time to exercise. σ is the volatility of the underlying asset.

When it comes to pricing chooser option, it should be adjusted as follows. First, this paper uses the Monte Carlo simulations, a numerical computation technique that employs plenty random samples to solve mathematical proble, to stimulate stock price S_U at time U. (The process is shown in Python the link is on page 4.) Second, this paper uses S_U to replace S_0 in the normal BS model to calculate the prices of call options and put options at choosing time U:

$$C_U^{BS} = S_U \Phi\left(d_1\right) - Ke^{-r(T-U)} \Phi\left(d_1 - \sigma\sqrt{T-U}\right). \tag{14}$$

$$P_U^{BS} = Ke^{-r(T-U)} \Phi\left(\sigma\sqrt{T-U} - d_1\right) - S_U \Phi\left(-d_1\right). \tag{15}$$

Where

$$d_1 = \frac{\log\left(\frac{S_U}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-U)}{\sigma\sqrt{T-U}}. \tag{16}$$

Then, at time U, compare the payoff of the call option and put option, and choose the bigger one to get the value of chooser optionX_U: $X_U = \max\left(C_U^{BS}, P_U^{BS}\right)$. Finally, discount X_U from time U to time 0 to price the chooser option at time 0 is $X_0 = e^{-rU}E[X_U]$ →Monte Carlo Method.

3. Experiments and analyst

3.1. Experiment I: efficiency and accuracy of the Black-Scholes model

There are 2 main purposes of this experiment. The first is to explore the accuracy of the BS model. Next is to compare the time of running the Black-Scholes model with the $N-period$ binomial tree model in Python. In this experiment, simulation times of Monte Carlo (M) are the independent variables, and the time steps (N) are fixed to 100. There are 2 main methodologies used in this experiment. One is based on the Black-Scholes Model with Monte Carlo Simulation, where the price may fluctuate with M because varying the times of Monte Carlo simulations can lead to different outcomes by this method. The accuracy of the option price was analyzed, and the corresponding computational time was recorded as M increased. Another one is based on the Binomial Tree Method, where the computational time for it is compared against the time in the Black-Scholes model with Monte Carlo simulation in this experiment, where N is 100.

The Python code developed in this study has been made available on GitHub for reference and further examination [10].

The parameter values are adjusted to better align with real-world conditions, and are specified as follows. Initial stock price $S_0 = 10$, Strike price $K = 10$, Risk-free interest rate $r = 0.1$, Volatility $\sigma = 0.2$, Chooser option selection time $U = 0.5$ (in years), Maturity time $T = 1$ (in years), Number of simulations M values range from 1000 to 5000 (times), Number of time steps $N = 100$ (times).

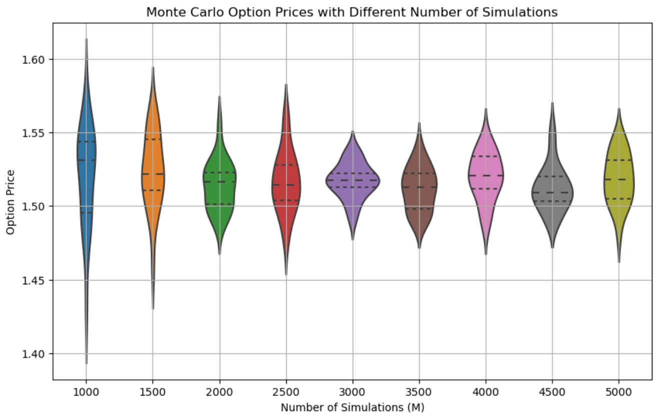

Figure 1: Monte Carlo simulation: impact of number of simulations on chooser option **pricing**

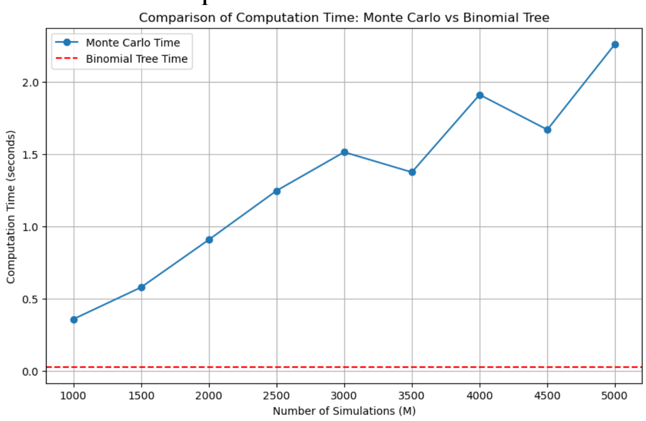

Figure 2: Comparison of computing time of Black-Scholes method and N-period binomial method

This section begins by showing violin plots in Figure 1 using pricing chooser option prices with the BS (Black-SCholes) model at different numbers of Monte Carlo simulations, representing accuracy by concentration of data. Then, it moves to compare the efficiency of the BS method and the N-period Binomial method, which are implied in the pricing chooser option by using a biquadratic chart in Figure 2.

Figure 1 presents a violin plot illustrating the relationship between option pricing determined by the Black-Scholes (BS) model and the number of Monte Carlo simulations conducted (denoted as M). Notably, as M increases, the distribution of option prices depicted in the plot exhibits a widening and a reduction in the height of the colored area. This change suggests a trend towards greater concentration of the data. Consequently, it can be inferred that the accuracy of the Black-Scholes method improves with an increasing number of simulations, thereby enhancing the reliability of the option pricing estimates derived from the BS model.

Figure 2 illustrates the computing time discrepancy between the BS model and the N-period binomial method. Since N is a constant in this experiment rather than an independent variable, the computing time associated with it remains independent of M. Consequently, this model appears as a straight, horizontal line (the orange line) on the graph, maintaining a consistent time of approximately 0.28 seconds. In contrast, the line representing the Black-Scholes model shows a fluctuating upward trend, which indicates that the computing time is directly proportional to the number of Monte Carlo simulations.

In conclusion, there is a trade-off between the accuracy and efficiency of the Black-Scholes method. From Figure 1, it can be concluded that an increasing number of Monte Carlo simulations enhances the accuracy of the Black-Scholes method. However, this increase leads to less efficiency since the running time is directly proportional to it, as shown in Figure 2. It is suggested that investors should carefully choose the M according to their needs.

3.2. Experiment II: convergence analysis of N-period binomial method and Black-Scholes method

The purpose of this experiment is to verify that as N gradually increases, the pricing results of the BS model and the binomial tree model gradually agree. In this experiment, the time steps (N) are the independent variables, and the simulation times of Monte Carlo (M) are fixed at 10000.

The chooser option in this experiment is priced by employing the Black-Scholes formula with Monte Carlo simulation, where the times of Monte Carlo simulation is 10000. To compare, an $N-period$ binomial tree model is implemented to price the chooser option for various N.

The calculations are performed in Python, and the results are presented in Figure 3.

The parameters are set as follows. Initial stock price: $S_0 = 10$, Strike price: $K = 10$, Risk-free interest rate: $r = 0.1$, Volatility: $\sigma = 0.2$, Chooser option selection time: $U = 0.5$ (in years), Maturity time: $T = 1$ (in years), Number of simulations: $M = 10000$, Number of time steps: N values range from 1 to 100. The calculations are performed in Python, and the results are presented in Figure 3.

In Figure 3, the horizontal axis shows the number of time steps, and the vertical axis shows the option prices. The two lines gradually stabilize at the 0.1 level and converge as N increases after the initial fluctuation (see Figure 3). This suggests that the option pricing results of the two methods are relatively consistent in this particular case, regardless of the variation in the number of time steps.

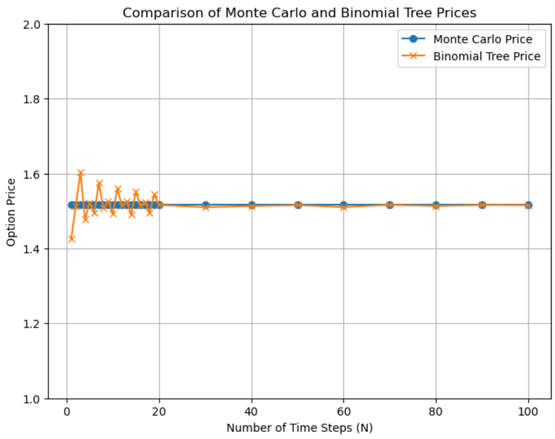

Figure 3: Comparison of Monte Carlo and binomial tree prices

In conclusion, this experiment demonstrates that the N-period binomial model converges to the price of the BS model for chooser options as the number of time steps is nearly infinite. This finding underscores the consistency and validity of the N-period binomial approach in approximating the theoretical pricing framework established by the Black-Scholes model.

4. Conclusion

Choosing options with flexibility is an invaluable tool for optimizing the benefits of the portfolio [11]. Investors can decide the direction of the option based on the latest market information and their judgment. Furthermore, it assists in managing uncertainty, providing investors with a greater opportunity to gather information and make more informed decisions in circumstances where market movements are challenging to anticipate. This essay has presented two methodologies for pricing chooser options: the N-period Binomial model and the Black-Scholes model combined with Monte Carlo simulations. This paper provides detailed analytical solutions for both approaches, along with their derivations, highlighting the distinct mechanisms through which each model operates. The experiment further investigated the accuracy and runtime of these two methods through Python, revealing important insights into their performance. However, limitations are also identified in the Black-Scholes model, like the trade-off between balancing efficiency and accuracy. Additionally, assumptions regarding the behavior of the Binomial model, specifically whether its runtime experiences an increase of N, are still unproven.

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Ming Ding and Jiaxin Liu contributed equally to this work and should be considered co-first authors.

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