

Black-Scholes Model and Its Application in the Insurance Industry

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Abstract: The Black-Scholes model, originally designed for short-term pricing of financial derivatives, has been explored as a quantitative method for the insurance industry for the evaluation of risks and premiums. In this paper, the usage of the model is explored systematically for investment-linked life insurance, catastrophe insurance, weather derivatives, and automobile insurance, analyzing its usefulness and limitations. Despite the model providing a formalized pricing framework, its assumptions of no market friction, volatility as a constant, and short-term application become challenging when the model is extended for long-term insurance contracts. According to the literature, the model can be further improved by modifying the volatility factors, including empirical findings, and using other stochastic models. Focusing on this aspect of derivative pricing in insurance, this research emphasizes the usefulness of finance derivative pricing principles for the insurance industry leading to product development and improved risk management as well as emphasizing the interdisciplinary relevance of finance to the insurance industry. While data nuances may not be applicable, further work is expected in making the novel model applicable for long determinants for various regulatory needs and for market-specific considerations such as health insurance, reinsurance, and insurance-linked securities.

Keywords: Black-Scholes model, Insurance industry, Investment-linked life insurance, Catastrophe insurance

1. Introduction

Theories for pricing financial derivatives have been crucial in shaping modern markets, with the Black-Scholes (B-S) model being the most prominent [1,2]. Derived from a geometrically Brownian motion process and Ito's lemmas, the model presents a mathematical basis—through the European option pricing model—for calculating the value of financial assets in a scenario of uncertainty. The model has performed well for a wide range of assets, from equities and bonds to foreign exchange products. Even though the B-S model was originally designed for short-term financial derivatives, the implementation of the B-S model for the insurance market has gained growing attention from scholars, particularly for the pricing of long-term risk products [3].

Insurance products increasingly assume the structure and exposure of financial derivatives. Investment-linked insurance guarantees, catastrophe insurance, weather derivatives, and new car insurance policies, for example, all involve market-based risks and contingent payments on outside occurrences [4,5]. Insurers, here, become, in effect, writers of options, offering minimum returns or assuming extreme losses. This resemblance has led the application of the tools of the world of

finance—namely the B-S model—to the insurance industry. But some complications exist when the application is translated from the world of finance into the world of insurance.

The B-S model relies on strong assumptions: log-normal distributions of prices, volatility that is constant, and frictionless markets [1]. In contrast, insurance markets involve long horizons, regulatory frictions, and exposure to extreme, typically non-normal events [6]. Products like catastrophe insurance or weather derivatives involve risks that are jumpy, seasonal, and mean-reverting—qualities incompatible with B-S assumptions. Even automobile insurance pricing, with the emergence of autonomous vehicle technologies, is subject to behavior and regulatory aspects outside the B-S model [4].

Several of these proposed modifications of the B-S framework for increasing its applicability for insurance pricing involve volatility parameter adjustment, the addition of empirical market observations, and the application of other stochastic processes [3,5]. While these attempts aim at maintaining the analytical tractability of the model, they also complicate its implementation and question its predictive power. Despite these ongoing efforts, there is no clear consensus on the effectiveness of the B-S model, even with modification, for describing the complexity of insurance pricing, especially for long horizons [6,7].

Due to the growing convergence of finance and insurance, there is a need for further exploration of the practical applicability of the model, validation and testing with empirical data for suggested improvements, along with hybrid approaches that can encompass both actuarial and finance frameworks. This article delves into the application of the B-S model in insurance, examining its limitations as well as the opportunities for further enhancement in insurance pricing, making it more dynamic and efficient.

2. Literature review

The old price-fixation methods were not sufficient to address the risk due to nature of product and increasing complexity with advancement in the insurance market as well in insurance products and their volatility. The Black-Scholes (B-S) model was initially developed to price financial derivatives and since then has been investigated in terms of its ability to price other insurance products owing to its mathematical structure [1,3].

The B-S model is also used for investment-linked life insurance (ILLI), particularly for the pricing of Guaranteed Minimum Death Benefits (GMDB), by treating GMDB as a put option, with the insurer being exposed to the downside risk. Despite being effective for the estimation of the cost of the guarantee, the constant volatility assumption is a serious flaw when the model is generalized for long-term insurance markets. Some authors recommend the volatility parameter adjustment, but others argue that such adjustment destroys the analytical tractability of the model [4].

The B-S model has also been utilized for catastrophe insurance, which covers catastrophes such as earthquakes and hurricanes, for the modeling of the trigger-based payments like the knock-out options. However, due to the non-normal distribution and jump nature of catastrophes, scholars suggest the incorporation of fat-tailed distributions or stochastic volatility for making the model appropriate [5,6].

Derivatives on weather, of critical importance for energy and agriculture, also possess the structure of financial options. The B-S model provides a starting point, but the seasonal and mean-reverting nature of weather variables is incompatible with the geometric Brownian motion upon which it relies. Consequently, stochastic mean-reverting processes and other advanced econometric approaches have been suggested as superior alternatives [7].

Within car insurance, particularly with the introduction of autonomous vehicle technology and usage-based insurance (UBI), the B-S model has also been thought of as a dynamic pricing mechanism. By incorporating accident probabilities and driver behavior, the model can adjust

premiums real-time. But due to the long-term nature and the presence of numerous non-monetary factors of risk, some believe that advanced actuarial models would be superior [4,8]. For example, in order to calculate the insurance rate of an insurance company in Heyuan City, Guangdong Province in 2022, people can first select 99,124 auto insurance policies written by his company in 2022, then arrange the data by dollar amount to get a chart shown in Table 1. After that, by fitting and forecasting the loss amount, and determining the parameters risk-free interest rate, executive price, policy value and volatility in the B-S model, the insurance rate is finally calculated [9].

Table 1: Economic losses and descriptive statistics of autonomous vehicle traffic accidents [9]

Chart B	N	Minimum	Maximum	Median	inequality	Standard Deviation	Skewness	Kurtosis
1-8	1200	1200	1350	1360	1374	1490	1500	1500
9-16	1500	1850	1866	1941	1956	2015	2091	2160
17-24	2760	2788	2920	2930	3000	3000	3000	3113
25-32	3150	3199	3271	3331	3338	3670	3739	3830
33-40	3850	3860	3885	3950	4200	4372	4488	4500
41-48	4500	4534	4590	4600	4611	4770	4774	4822
49-56	5000	5000	5023	5245	5291	5470	5600	5639
57-64	5668	5900	5957	6001	6156	6348	6760	6878
65-72	7064	7264	7394	7406	7740	7887	7923	7961
66-80	8194	8359	8848	9659	9742	9810	9960	10105
81-88	10519	10650	10838	11089	11542	12345	12509	13680
89-96	13931	14269	14297	14329	15921	16738	17284	18066
97-104	18517	18776	20559	20913	21452	23269	23707	24577
105-112	27400	27494	27925	35033	35318	38970	41113	44982
113-120	49846	57327	59139	65842	84186	98712	103631	142820
X	120	1200	142820	5957	14203	21761	3.445	13.949

The B-S model has also been utilized for the estimation of longevity risk with the help of securitization methods. Despite being supportive of mortality volatility and risk-free rate-based pricing, data insufficiency and long-term demographic uncertainty limit its application. Incorporation of the model with stochastic or non-deterministic frameworks is proposed by some scholars for greater applicability of the model for real-life longevity examples [9,10].

3. Methodology

3.1. Theoretical model overview

A systematic literature review was conducted to explore the Black-Scholes model applicability in the insurance industry. This paper reviews existing academic literature on the B-S model and its application in different forms of insurance products to evaluate the consistency of its theoretical foundations with the dynamics of the insurance market. Then, the author assesses the assumptions of the B-S model and its suitability for such products as life insurance with guarantees, catastrophe insurance, weather derivatives, and auto insurance. Mathematical formulations are integrated to demonstrate how the model quantifies risk and informs pricing strategies.

Mathematically, the B-S model is grounded on Geometric Brownian Motion (GBM), representing the stochastic progress of asset prices:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \quad (1)$$

where S_t represents the asset price at time t , μ is the drift rate, σ is the volatility, and dW_t denotes a Wiener process. This equation assumes that asset prices follow a lognormal distribution over time. In the insurance sector, the B-S formula is applied to pricing insurance products with financial derivative-like structures, such as investment-linked life insurance (GMDB) and catastrophe insurance.

In insurance contracts that resemble financial options, the Black-Scholes option pricing formula is commonly applied to determine the theoretical value of payouts. The formula for a European call option is expressed as

$$C = S_0 N(d_1) - K e^{-rT} N(d_2) \quad (2)$$

Here, C represents the option price, S_0 is the current asset price, K is the strike price, r is the risk-free interest rate, T is the time to maturity, and $N(d)$ is the cumulative normal distribution function. In the insurance industry, this formula is often adapted to price GMDB and assess catastrophe insurance risks. However, given the long-term horizons of insurance markets and the occurrence of extreme events, some assumptions in the B-S model may require modifications to better capture the risk dynamics specific to insurance contracts.

The first steps undertaken in the research process are as follows. First, a literature review is conducted by gathering and analyzing research that describes the use of the B-S model for pricing insurance contracts. This step aims to list all known applications and adaptations of the model for various types of insurance products. Second, mathematical modeling is employed to verify the compatibility of the primary equations of the B-S model in relation to the risk profiles of different insurance products. This paper tests the validity of these assumptions, particularly as they relate to non-regression events and distributional risks in the insurance sector. Third, the article attempts to break down such traditional conservatism with widely different research perspectives and applies the advantages versus disadvantages appraisal of B-S model to insurance pricing. This work also amends several points of the model, such as setting an expectation about volatility and changing the movement randomness by choosing non-gaussian distributions for price changes, in order to increase its adaptability into insurance.

3.2. Significance of the model in insurance

The B-S model offers a mathematical tool for pricing insurance products, akin to financial derivatives, such as, but not limited to, insurance-linked life instruments (ILLI), cat insurance products, weather derivatives & auto products. Although this model was originally designed to target short term financial derivatives, it has also been applied in the insurance pricing domain as it measures risk for market-based products [11]. However, the assumptions adopted, such as constant volatility and normally distributed prices, often are not consistent with the way long-term insurance contracts behave, so it is not without challenges.

GMDB are priced using the B-S model in ILLI, assuming that they are essentially put options. While the assumption allows insurers to estimate the cost of guarantees, the constant volatility assumption fails to match the dynamic nature of the insurance markets. A few academics do try changing the volatility parameters but then the model becomes much more complex without proving to be more accurate.

This has fundamental problems in catastrophe insurance because the risks involved are extraordinarily fat tailed, thus the B-S model that assumes normal distributions must be used. This has led to the proposed modifications of stochastic volatility and fat-tailed distributions. Nevertheless, some scholars maintain that catastrophe risks, stemming from natural disasters, need to be modeled with non-financial methods, meaning no B-S adaptation can faithfully account for these dynamics.

The model assumes that returns are normally distributed, which does not hold in reality, especially for weather variables. To improve the model applicability, stochastic mean-reverting processes have been suggested. It shows that with creative application, the financial theory has its place in insurance pricing as well. But then, its principles usually require adjustment for the different nature of the insurance business.

4. Results

The findings in this paper provide new insights regarding the application of the B-S model as a pricing instrument in the insurance field for many products that behave like financial derivatives. The literature review and mathematical analysis further demonstrate the generalization of the framework observed in the model to many other insurance products, such as investment-linked life insurance, catastrophe insurance, weather derivatives, and automobile insurance. Nonetheless, the B-S model is still being developed practically, and its application must be continuously improved.

The B-S model is a standard model for pricing options (and other derivative securities) and can be used as a basis for pricing insurance products related to financial options. In this sense, the model is applied in the context of GMDDB to quantify the liability of the insurer as the market moves, which is analogous to pricing a put option. The mark-to-market values associated with the underlying assets are crucial for integrating the respective credit risk. The B-S framework helps insurers measure financial risks, resulting in the value of guaranteeing the lowest payments to every insured, which can be used to calculate appropriate premium charges. However, market volatility is highly variable over long time horizons, and thus the model's baseline assumptions need to be adjusted. To better fit real-life conditions, it is crucial to modify the parameters for volatility.

The B-S model has been utilized for catastrophe insurance to estimate premiums by characterizing payout structures as barrier or knock-out options. Its mathematical structure is appropriate for quantifying exposures for trigger-based payments. However, literature suggests that the application of common distributions should be modified because catastrophes are extreme and non-normal. Despite the B-S model providing a strong framework for estimation, additional adjustments—such as the addition of fat-tailed distributions or stochastic volatility—must be performed to account for the inherent unpredictability of large catastrophes. Empirical observations tend to show significant disparities between theoretical prices and observed claims, further supporting the need for context-specific calibration.

Its application to weather derivatives is also well-documented, with several studies showing the effectiveness of the model for pricing products with weather variables such as temperature and precipitation. These products, however, display behaviors opposite to those assumed by the model, such as seasonality and mean reversion. Some authors suggest that the model should be viewed more as a framework, requiring better stochastic processes to describe weather behavior. These findings suggest that direct application for climate finance will involve significant parameter adjustments. In practice, hybrid models that combine climatological data with mathematical finance prove stronger and better suited to insurer needs.

In the context of auto insurance, innovations such as autonomous driving and usage-based insurance (UBI) have enabled dynamic risk assessment models. The B-S model has been proposed as a framework for real-time adjustment of accident probabilities and premiums. Its adaptability supports emerging pricing mechanisms, especially those individualized based on risk. However, because auto insurance is long-term, the model's parameters need periodic updates to remain relevant. Non-market factors like driver behavior, infrastructure, and regulations also present challenges for purely financial modeling. Interdisciplinary approaches that integrate actuarial science, behavioral analysis, and technology are necessary.

The results show that the B-S model is valuable for modeling volatility and structured payment item insurance within the market. It is observed that each part of the insurance market has its own problems, requiring the model framework to be adapted. Conceptually strong, the model will be refined and tested for real-world relevance. While the model has proven conceptually strong, practitioner experience indicates that fulfilling its operational function will rely on overall simplicity, legal compliance, and adaptability of the data involved, all factors that may influence its success. As insurers' product offerings become more complex and integrated into broader financial and environmental systems, the model's flexibility will be critical to its long-term relevance. Further refinements will be required to close the gap between theory and practice.

5. Conclusion

This paper classifies and manages most of the applications over the insurance sector of the B-S model, especially concerning GMDB, catastrophe insurance, weather derivatives and automobile insurance. The B-S model delivers a quantitative and methodological framework to facilitate the pricing of insurance products with characteristics resembling the mechanics of financial derivatives, and short-term options in particular. The model is applicable to products such as GMDB and weather derivatives, whose structures resemble financial options closely. These apps enable insurers to effectively evaluate risks and determine premiums. However, applied to long-term insurance contracts or highly volatile risks, such as catastrophe and automobile insurance, the assumptions of the model become less appropriate. The problem stems from the model's assumptions — frictionless markets, constant volatility, short-term applicability — being mismatched with the realities of most insurance markets that are complex, long-term affairs.

However, the B-S model has helped greatly in the evolution of quantitative pricing techniques for insurance despite its flaws. Researchers have suggested multiple adaptations to address its limitations. The two main options were introducing stochastic volatility, modifying the parameters of volatility or real-world data. These changes do have the potential to increase the model performance, but they also make the model much more complex without necessarily improving the quality. The tension between simplicity and improved predictive accuracy is still a contested topic of discussion. Furthermore, the model is ill-suited for extreme, non-normally distributed risks — seen in fields like catastrophe insurance — the kinds of risks that hold the most danger. Catastrophic events typically have jump type dynamics which the B-S model, utilizing normal distributions cannot properly characterize.

As such, the effective use of the B-S option pricing model must be customized for the insurance industry. Further research is needed to explain more detailed phenomenon for other types of life insurance markets (e.g., using nonnormal distributions and more elaborate risk models), with a focus on constructing more sophisticated variants of the same model that can remain solvent in the long term. Methods include stochastic volatility models, machine learning based pricing and hybrid approaches involving financial and actuarial methods. Wider applicability of the model to other insurance products, including health insurance, reinsurance, and insurance-linked securities, can enhance its utility. Overall, though it has shown its worth in some areas, the B-S model can only be effectively utilized in the insurance market given sufficient research and adaptation to the specific context.

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