Proceedings of CONF-MPCS 2025 Symposium: Leveraging EVs and Machine Learning for Sustainable Energy Demand Management
DOI: 10.54254/2753-8818/2025.GL23430

Research on the Economic Growth Models and Their Mathematical Principles

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Abstract: This study embarks on a comprehensive journey into the intricate and multifaceted world of economic growth models, with a particular emphasis on the mathematical principles that underpin and drive them. At its very core, it meticulously examines the Solow Growth model, a fundamental and highly influential theory that has long been a cornerstone in the realm of economics. Through the sophisticated application of differential equations, the Solow model constructs a dynamic and powerful framework designed to analyze the long - term evolution of economies with remarkable precision. Capital accumulation, which encompasses the gradual process by which an economy builds up its stock of physical capital, such as machinery and infrastructure, as well as human capital through education and training, is identified as a key driver of growth. Meanwhile, labor or population growth supplies the necessary workforce for production, ensuring the continuous operation of economic activities. Technological progress, often considered the most transformative and game - changing factor, continuously enhances productivity by introducing new methods, tools, and knowledge. Together, these elements are not only crucial for gaining a deep - rooted understanding of the underlying mechanisms of economic growth but also for formulating well - informed policies that can effectively foster sustainable development in the long run.

Keywords: Economic growth model, mathematical principle, Solow growth model.

1. Introduction

Economic growth models are mathematical representations or theories that describe the mechanisms driving the growth of an economy. They typically examine changes in GDP or other measures of aggregate economic activity over time. These models help explain how various factors-such as capital accumulation, labor force expansion, technological progress, and policy interventions-contribute to economic growth.

The origins of economic growth models can be traced back to the classical economists of the 18th and 19th centuries. Thinkers like Adam Smith, David Ricardo, and Thomas Malthus laid the foundation for understanding economic growth. Adam Smith, in the seminal work The Wealth of Nations, emphasized the role of specialization, the division of labor, and trade in increasing productivity. Smith argued that expanding markets and improving efficiency were key drivers of economic growth. Similarly, David Ricardo introduced the concept of comparative advantage, highlighting how trade and the efficient allocation of resources could enhance overall economic output. However, not all early thinkers were optimistic-notably Thomas Malthus predicted that population growth would outpace food production, leading to stagnation. This pessimistic view, known as the Malthusian trap, was later challenged by technological advancements during the Industrial Revolution [1-3].

During the mid-20th century, the focus shifted to neoclassical growth models, which formalized economic growth theories with mathematical precision. The Solow-Swan Growth Model, developed independently by Robert Solow and Trevor Swan in 1956, marked a significant breakthrough. This model introduced the concept of a steady state, where an economy grows at a constant rate determined by technological progress and population growth. The key insight of the Solow model was that capital accumulation alone could not sustain long-term growth due to diminishing returns to capital. Instead, technological progress, treated as an exogenous factor, was identified as the critical driver of sustained growth. The model's assumptions, including constant returns to scale and diminishing marginal returns to capital, provided a foundational framework for understanding growth dynamics [4].

In the 1980s, the limitations of neoclassical models-particularly their treatment of technological progress as an external factor-led to the development of endogenous growth theories. These models, pioneered by economists like Paul Romer and Robert Lucas, integrated innovation, knowledge creation, and human capital directly into the growth process. Paul Romer's model emphasized that knowledge and technological advancements arise from economic activity and investments in research and development [5]. This marked a departure from earlier models by showing that growth could be influenced by deliberate policy decisions. Similarly, Robert Lucas focused on the role of human capital and education in fostering long-term growth. Endogenous growth models highlighted the importance of government policies, such as subsidies for education and R&D, in driving sustained economic expansion.

More recently, unified growth theory has emerged as a framework for understanding the transition from historical stagnation (as seen in the Malthusian era) to modern sustained growth. This approach integrates insights from both classical and modern growth models to explain the long-term evolution of economies. By bridging historical and contemporary perspectives, unified growth theory seeks to provide a comprehensive explanation of economic development across different stages of human history.

2. Literature review

Economic growth models are needed to simplify the complex reality of economies so that outcomes can be predicted. This is essential for governments to make good policies and businesses to employ profitable strategies, to the best of their ability.

The Solow-Swan Growth Model was first introduced by Solow, which responded to and built on earlier economic growth models such as the Harrod-Domar model. In this paper, Solow formalized how capital accumulation, labor growth, and exogenous technological progress drive economic growth. Solow demonstrated that diminishing marginal returns to capital lead to a steady state of growth where capital and output grow at the same rate as the labor force, and the capital-labor ratio grows due to technological progress. This result showed that the economy can converge to a steady growth path in the long run, instead of the instability predicted by the Harrod-Domar model [1].

However, the absence of human capital in the Solow-Swan Growth Model is addressed in Mankiw et al. [2]. This paper highlighted how the observed data shows that the impacts of saving and population growth on income are larger than the model suggests, and to address this, the authors incorporate human capital into the model. The results show that the augmented model better explains cross-country variation in income per capita, accounting for almost 80% of the variation.

Another limitation of Solow's model is the treatment of technological progress as exogenous, which is critiqued in Jones. The Solow model assumes that technological progress occurs independently of economic factors such as R&D expenditure and education, which is an

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oversimplification. This paper discussed how endogenous growth models build on Solow's foundation by incorporating R&D and innovation. R&D-based models consider how technological progress results from intentional investments in research and development activities, which are influenced by economic factors [3]. This emphasizes the importance of policy to foster innovation and education, increasing the technological progress and therefore the growth rate of the economy. The assumption of a closed economy is another limitation of the Solow-Swan Growth Model. Barro extended the Solow model to incorporate trade openness as a driver of productivity to analyze the impact of policy reforms and trade openness on economic growth. The authors argued that trade liberalization leads to an increase in the steady-state level of income by enhancing technological diffusion and efficiency [4].

The predictions of Solow's model are tested against cross-country data in Islam. This paper found that the convergence hypothesis with poorer countries growing faster than richer ones is only supported when human capital is held constant. Other findings include that government consumption is inversely related to growth, while public investment shows little correlation to growth [5]. However, Sachs et al. started with the subject instead of the reference symbol advocates for a panel data approach to study growth convergence instead of the cross-country approach used in Islam [5, 6]. The panel data model accounts for the influence of past values on current outcomes and country-specific effects which may not be observable, allowing for a more reliable analysis. This paper showed how different countries may converge at different rates depending on country-specific effects and initial conditions, providing robust evidence for conditional convergence across countries.

Acemoglu provided a comprehensive overview of growth models including the Solow-Swan Growth Model, discussing its relevance and critiques [7]. The Solow model provides a foundational framework for understanding how capital accumulation, labor and technological progress affect economic growth. The model highlights the importance of saving rates and technological progress for long-term growth, which can be used for policymakers to achieve a desirable growth rate. However, the Solow Growth Model is an oversimplification due to the absence of human capital and the assumption of exogenous technological progress [8, 9]. With newer endogenous growth theories that highlight the roles of factors like innovation and institutions, the Solow-Swan Growth Model has become less relevant.

3. Main findings

3.1. Initial equations

The Solow growth model models the relationship over time between capital, labor and technological growth using differential equations, and shows how these factors affect output growth [10].

The labor force $L(t)$ grows at a constant rate n: $\frac{dL(t)}{dt} = nL(t)$. The author can solve this differential equation: No first-person pronouns should appear throughout the text:

$$\int \frac{\mathrm{d}L(t)}{L(t)} = \int n\,\mathrm{d}t \tag{1}$$

$$ln\big(L(t)\big) = nt + c \tag{2}$$

$$ln\left(\frac{L(t)}{k}\right) = nt \tag{3}$$

$$L(t) = k\mathrm{e}^{nt} \tag{4}$$

For this equation to represent growth, k must be the initial labor force. Therefore $L(t) = L(0)e^{nt}$ where $L(0)$ is the initial labor force.

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Technology level $A(t)$ is modelled similarly, so $\frac{dA(t)}{dt} = gA(t)$ and $A(t) = A(0)e^{gt}$ where g is the constant rate of technological growth.

The output $Y(t)$ is described using a Cobb-Douglas production function: $Y(t) = A(t)K(t)^a L(t)^{1-a}$, where a is the capital share of output (fraction of output which comes from capital), $K(t)$ is the capital stock.

The capital accumulation equation accounts for investment, which adds to the capital stock, and depreciation of capital, which could be due to wearing out or technological advancements.

Therefore, the equation is: $\frac{dK(t)}{dt} = sY(t) - \delta K(t)$, where s is the savings rate, Investment is a fraction (s, the savings rate) of output Y(t), δ is the depreciation rate of capital.

3.2. Steady-state capital and output per worker

In the Solow growth model, the steady state is where the economy has reached a point where investment in new capital is cancelled out by depreciation and the dilution of capital due to labor force growth, meaning capital per worker $k(t)$ stops growing.

$$Capital\ per\ worker\ = k(t) = \frac{K(t)}{L(t)} \tag{5}$$

Capital accumulation equation is $\frac{\mathrm{d}K(t)}{dt} = sY(t) - \delta K(t)$. So $\frac{\mathrm{d}k(t)}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{K(t)}{L(t)}\right) = \frac{L(t)\frac{\mathrm{d}K(t)}{\mathrm{d}t} - K(t)\frac{\mathrm{d}L(t)}{\mathrm{d}t}}{L(t)^2}$, $\frac{\mathrm{d}k(t)}{\mathrm{d}t} = \frac{L(t)\left(sA(t)K(t)^a L(t)^{1-a} - \delta K(t)\right) - K(t)nL(t)}{L(t)^2}$, keep the font size of the formula consistent:

$$\frac{\mathrm{d}k(t)}{\mathrm{d}t} = sA(t)\frac{K(t)^a}{L(t)^a} - \delta\frac{K(t)}{L(t)} - n\frac{K(t)}{L(t)} \tag{6}$$

$$\frac{\mathrm{d}k(t)}{\mathrm{d}t} = sA(t)k(t)^a - (\delta + n)k(t) \tag{7}$$

To derive the steady-state capital per worker this paper sets $\frac{dk(t)}{dt} = 0$. Set steady-state capital per worker be k^*.

$$0 = sA(t)(k^*)^a - (\delta + n)k^* \tag{8}$$

$$(\delta + n)k^* = sA(t)(k^*)^a \tag{9}$$

$$(k^*)^{1-a} = \frac{sA(t)}{\delta + n} \tag{10}$$

Steady-state capital per worker is $k^* = \left(\frac{sA(t)}{\delta+n}\right)^{\frac{1}{1-a}}$. This reflects how efficiently capital is distributed relative to labor. To find the steady state output per worker, first let output per worker = $y(t) = \frac{Y(t)}{L(t)}$.

Production function is $y(t) = \frac{Y(t)}{L(t)} = \frac{A(t)K(t)^a L(t)^{1-a}}{L(t)}$, where $y(t) = A(t)k(t)^a$. Therefore, the steady state output per worker is $y^* = A(t)\left(\frac{sA(t)}{\delta+n}\right)^{\frac{a}{1-a}}$.

The above formula parts are somewhat piled up. Some necessary textual explanations need to be added to connect with the theme of this article and reflect the complete analytical logic and the author's own thoughts

4. Results and discussion

4.1. Importance of the findings

Capital per worker and output per worker act as indicators of economic performance. Governments can aim to increase these by encouraging technological progress, such as by funding research institutions, or increasing the savings rate, possibly by promoting tax-free savings accounts.

The steady state capital and output per worker formulae show that capital per worker converges in the long run while output per worker will grow at the rate of technological growth. Predicting the long run behaviour of the economy using steady state allows governments to set realistic goals. It also helps guide policies as if the economy is not in steady state, it might suggest the need for policy changes such as adjusting saving rates or technological investments in order to achieve a desired growth path. The formulae also highlight how higher population growth reduces capital per worker because capital gets diluted across more workers.

4.2. Limitations

The Solow growth model assumes a closed economy, meaning no international trade or foreign investment. These factors can affect capital accumulation and technological progress. Acemoglu shows that by extending the model to include trade openness as a driver of productivity, that global trade can increase the steady state capital and output per worker of an economy [7].

The Cobb-Douglas production function is assumed, but different countries will have different systems of production.

Technological progress is treated as an exogenous factor, meaning the model does not explain how technology improves, but instead it just grows at a constant rate g. R&D-based models which treat technology endogenously can solve this problem, as discussed in Jones [3].

Human capital such as skills and education are not included in the model as a factor of production, but human capital increases labor productivity and therefore capital and output per worker. Mankiw demonstrated that incorporating human capital into the Solow Growth Model much better explains cross-country variation in income per capita [2].

The constant depreciation rate of capital is not realistic as in some economies with rapid technological growth, old capital depreciates more quickly. The model does not include the role of government policies such as taxation, which can heavily affect investment and savings.

5. Conclusion

Overall, the utilization of differential equations in the Solow Growth model serves as an exceptionally powerful analytical tool, enabling economists to comprehensively understand the intricate and multifaceted relationships between various economic factors. By incorporating these equations, the model can precisely capture the nuanced ways in which alterations in elements such as savings rates and technological advancements influence the complex process of capital accumulation. For instance, a higher savings rate not only directly translates into increased investment but also has a compounding effect over time. This additional investment funds the purchase of new machinery, construction of infrastructure, and training of the workforce, all of which accelerate capital accumulation and enhance an economy's productive capacity.

Moreover, this sophisticated mathematical approach equips economists with the means to predict economic behavior in the long run with remarkable accuracy, especially regarding key metrics like capital per worker and output per worker. Through a meticulous examination of the steady - state conditions of the model, economists can determine the equilibrium levels of these crucial variables. This valuable insight serves as a guiding light for governments, providing them with a well - defined

DOI: 10.54254/2753-8818/2025.GL23430

model growth path to strive for. It empowers policymakers to formulate effective economic policies, allocate resources in a more efficient manner, and ultimately drive sustainable development that benefits society as a whole.

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