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Fourier Transform for Seasonality Detection in Corporate Revenue (and Operating Cash Flow)

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Abstract: This study examines the application of Fourier transforms in enhancing financial forecasting models for the automotive industry. This study focuses on the top five global automakers (Toyota, Tesla, Ford, Volkswagen, and Mercedes-Benz) and analyses quarterly revenue data from 2015 to 2024. These 5 companies together account for approximately 20.6% of the global automotive industry's total turnover. As a result, their average quarterly sales may reflect some aspects of the financial situation of the automotive industry as a whole. So, this study will not only analyse their individual quarterly revenues, but will also focus on how they, as part of the industry as a whole, create this kind of value. ARIMA is used in this study to predict and analyze the quarterly revenue of these 5 companies in this study, as it is recognized that traditional ARIMA (1,1,1) models have the capacity to capture trends and short-term dynamics. However, they cannot frequently identify underlying and irregular seasonality. Fast Fourier Transform (FFT) is a method frequently used by researchers to detect hidden and irregular seasonal patterns, which can significantly improve forecasting accuracy. As a result, this study will propose the application of FFT to extract dominant cyclical patterns and integrate them into a Fourier-augmented ARIMA model.

Keywords: Fourier Transform, ARIMA, automotive industry, seasonality.

1. Introduction

Studies [1] indicate that the financial performance of the automobile industry has great seasonality and cyclical variations due to factors like market demand, new model launches, government incentives, and broader economic trends. While traditional forecasting models like ARIMA have tried to describe the periodic patterns, there are still huge limitations, especially when seasonal effects are irregular or influenced by external shocks such as supply chain disruptions or policy changes.

Fourier transform provides an accurate way to decompose time series into a frequency part. This can help get rid of noise. Recent studies [2] have shown that using Fourier Transform with other forecasting models can improve its prediction accuracy. For instance, Tesla's sales often increase at the end of each quarter, while Toyota and Volkswagen have more steady sales. Understanding these cyclical patterns can absolutely improve demand forecasting and financial planning.

So, this study aims to investigate the application of the Fourier transform in financial forecasting within the automobile industry in revenue and cash flow. The research will focus on five major automotive companies—Volkswagen, Toyota, Ford, Mercedes-Benz, and Tesla—analyzing their historical financial data to identify common industry trends. Additionally, this study will explore the financial differences between internal combustion engine (ICE) vehicle manufacturers and electric vehicle (EV) manufacturers, as well as the impact of different supply chain models (dealership-based vs. direct sales) on financial cyclicality.

2. Literature review (or theoretical background)

2.1. Limitation of ARIMA in seasonal financial data

ARIMA (AutoRegressive Integrated Moving Average) is one of the most widely used time series prediction models, and it is expressed by ARIMA (P, D, Q).

- P is the order of the autoregressive model
- D is the degree of differencing
- Q is the order of the moving-average model

Given the moderate sample size and the aim for model prediction, ARIMA (1,1,1) is adopted as a baseline in this essay. Its function is:

$$y_t = \mu + \emptyset_1 y_{t-1} + \theta_1 \epsilon_{t-1} + \epsilon_t \quad (1)$$

- y_t is the current value of the time series
- ϵ_t is the random error at time t.
- $\emptyset_1 y_{t-1}$ shows the influence of the previous value y_{t-1}.
- $\theta_1 \epsilon_{t-1}$ shows the influence of the previous error ϵ_{t-1}.

It relies on the smoothness of the data series, assuming a linear relationship. When dealing with seasonal movements that are non-regular or subject to external shocks, its accuracy will decrease a lot. Studies [3] have pointed out that ARIMA has problems analyzing car sales, which can be affected by outside shocks like supply chain problems, external policy, and many other events.

2.2. Fourier transform

Fourier transform decomposes a time-domain signal into a frequency-domain function. It can show the periodic structure of the signal by using sinusoidal functions. Fourier Transform can further analyze the periodicity of the data and reduce RMSE by isolating seasonal patterns [3].

In financial prediction, the Fast Fourier Transform (FFT) is widely used, with a mathematical expression of:

$$y_t \approx a_0 + \sum_{k=1}^{K} [a_k cos(\frac{2\pi kt}{T}) + b_k sin(\frac{2\pi kt}{T})] \quad (2)$$

- T is the period length
- K is the number of frequency components to retain
- $a_k b_k$ are the coefficients of cosine and sine components
- a_0 is the constant term

This transform can be used to find the frequency and period of the most obvious seasonal change, which can lead to a better understanding of the time series data.

2.3. Fourier-based hybrid model

In recent years, researchers have started to combine Fourier-based preprocessing with traditional and deep-learning forecasting models. Thompson and Evans (2022) showed that combining FFT

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with ARIMA improved periodic detection and residual minimization. Conventional seasonal decomposition techniques rely on fixed calendar periods. Fourier analysis, in contrast, is able to flexibly identify hidden periodicities even when they are not strictly aligned with calendar quarters or months [4]. This is of particular relevance in the automotive industry, where patterns may shift due to product cycles, policy incentives, or global supply disruptions.

In addition, hybrid models that use FFT-preprocessed data often perform better than standalone models in terms of stability and ease of interpretation. For example, when the most critical seasonal patterns are found using FFT and then used as inputs into linear models or as extra features in LSTM networks, the models are better at focusing on the essential economic structures.

FFT can also make financial models more straightforward, which makes it easier to communicate with investors. Unlike black-box models that simply "fit" data, Fourier-enhanced frameworks help stakeholders understand regular business cycles, such as BYD's post-incentive revenue lags. As the study [5] explains, the fact that frequency-domain methods can be understood makes them particularly useful in regulatory and managerial contexts.

As a result, Hybrid forecasting models that use Fourier transformation are both practical and accurate. They are especially useful for industries that have complex and changing patterns.

By using Fourier Transform in the ARIMA:

$$y_t = \mu + \emptyset_1 y_{t-1} + \theta_1 \epsilon_{t-1} + \epsilon_t + a_k cos(\frac{2\pi kt}{T}) + b_k sin(\frac{2\pi kt}{T}) \tag{3}$$

3. Methodology (or materials and methods)

3.1. Data collected

Five major automotive companies' (Volkswagen, Toyota, Ford, Tesla, Mercedes-Benz) revenue is chosen for the following reasons:

1. Representativeness: These companies represent the global automotive landscape, including ICE leaders (Toyota, Ford), European premium brands (Volkswagen, Mercedes-Benz), and the most influential EV disruptor (Tesla).
2. Data quality: All five firms can provide reliable quarterly reports and can be found easily through financial websites like Macrotrends.
3. Global diversity: These companies cover Asia, North America, and Europe, leading to more general insights.
4. Distribution strategy: These companies can show diversity in sales channels and vehicle technologies

- Direct-to-consumer EV brand: Tesla
- Transitioning to direct sales ICE/EV brand: Mercedes-Benz, Volkswagen
- Dealership-based ICE brands: Toyota, Ford

This diversity can support a comparative analysis of how sales structure and vehicle electrification affect revenue's periodicity.

Quarter revenue data is chosen between 2015 and 2023 because it covers the important period of the global automotive industry's transition from ICE to EV and also includes the dramatic market and supply chain shocks caused by COVID-19.

Then, by using Macrotrends and Excel, primary data is collected, as shown in the figure below.

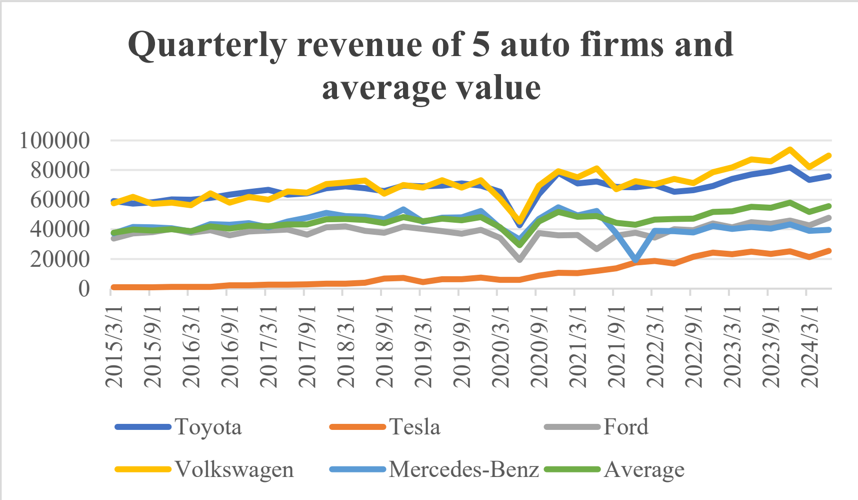

Figure 1: Quarterly revenue of 5 auto firms and average value

From figure 1, the seasonality is not clear. So, Fourier Transform will be used to further process these data.

3.2. Fourier transform

Although it's technically right to directly do Fourier transform on raw data, there are still some implicit requirements on the time series data. Without preprocessing, transform may be meaningless.

So, all the time series data must be preprocessed. Using Python, missing values are interpolated, extreme outliers are capped based on z-scores, long-term trends are removed by mean detrending, and data are aligned to equal quarterly intervals to ensure the mathematical validity of the transformation, which can be shown by figure 2.

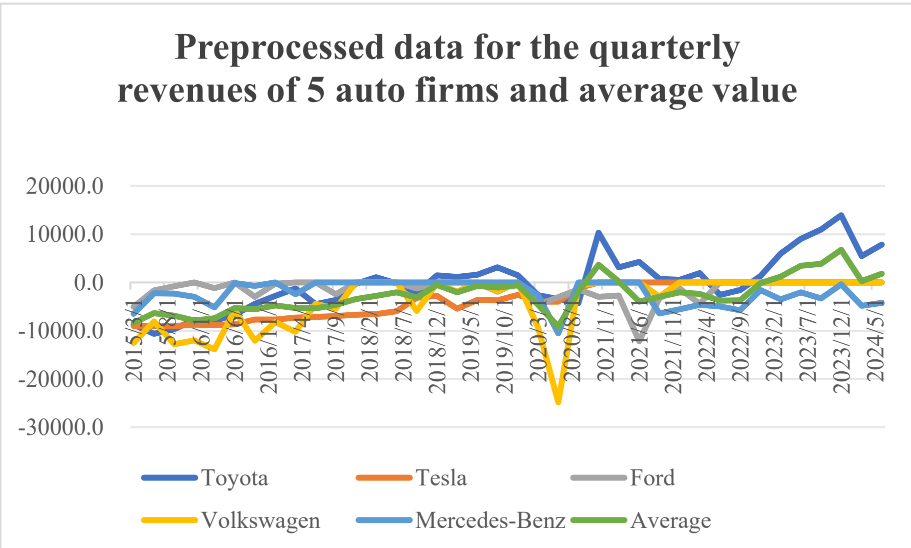

Figure 2: Preprocessed data for the quarterly revenues of 5 auto firms and average value

In order to decompose the companies' quarterly revenue y_t into a combination of cosine and sine functions, python is implemented by the following steps.

1. Get the target time series.

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2. Apply the Fast Fourier Transform by numpy.fft.fft().
3. Generate frequency value by fft.fftfreq().
4. Keep only the positive frequency.
5. Get a_k and b_k.

$$a_k = \frac{2}{\text{length of the time series}} * \text{Re(FFT)} \tag{4}$$

$$b_k = \frac{2}{\text{length of the time series}} * \text{Im(FFT)} \tag{5}$$

6. Use Matplotlib to plot the spectrum.

3.3. Forecasting model construction

Through the Fourier Transform of the data, the forecasting model ARIMA can be improved.

1. Difference the series

This step transforms the model from ARIMA (1,1,1) to ARIMA (1,0,1) in order to make the data stationary. This can be done by

$$\Delta y_t = y_t - y_{t-1} \tag{6}$$

2. Identify the dominant cycle

To recognize seasonality that is not caused by calendar quarter, the Fast Fourier Transform is applied.

The value of T can be measured, which is the number of quarters in the dominant cycle.

3. Use the period to generate the Fourier part, which is $\cos(\frac{2\pi t}{T})$ and $\sin(\frac{2\pi t}{T})$

4. Fit the ARIMA (1,0,1) + Fourier model and extract all the parameters

Finally, new, more accurate, and practical forecasting models can be made.

4. Result

4.1. Automobile industry's quarterly revenue

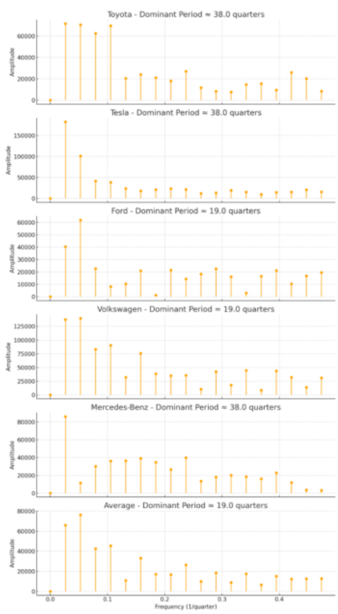

Figure 3: Preprocessed data for the quarterly revenues of 5 auto firms and average value

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Table 1: Dominant frequency and dominant period

Company	Dominant frequency	Dominant period
Toyota	0.0263	38
Tesla	0.0263	38
Ford	0.0526	19
Volkswagen	0.0526	19
Mercedes-Benz	0.0263	38
Average	0.0526	19

Figure 3 above shows the frequency-domain spectra for each of the 5 auto companies and their average value (representing the whole automobile industry). The x-axis is frequency, while the y-axis is amplitude, an indicator of each frequency's contribution to the quarterly revenue variation.

From table 1, it is obvious that Toyota, Tesla, and Mercedes-Benz have the same dominant frequency of approximately 0.0263, which is a cycle of 38 quarters (9.5 years). These longer cycles demonstrate that their revenue is driven by structural forces like platform transition and production scale-up.

By contrast, Ford and Volkswagen have a larger dominant frequency of approximately 0.0526, which is a cycle of 19 quarters (4.75 years). These shorter cycles demonstrate that their revenue is driven by reactive forces, like seasonal demand shifts and regional sales volatility.

The “average” part shows the whole automobile industry’s situation, which is taken by the mean of the quarterly revenue of the 5 firms. From the figure, the average part has a dominant frequency of approximately 0.0526, which is equal to Ford’s and Volkswagen’s. As a result, the industry is heavily shaped by Ford and Volkswagen, probably due to their larger yield, broader geographical locations, and more loyal customers.

By contrast, the other 3 firms smooth out the average process. Their long-run fluctuations do not dominate the aggregate spectrum. This means that the average signal reflects the mid-range cycle, not the long-term strategies of Toyota, Tesla, and Mercedes-Benz.

It can lead to several findings:

Average industry revenue cannot represent individual firms, especially those innovative and luxury automobile firms.

People should not just rely on the whole automobile industry’s data since firm-level heterogeneity is more important.

A 4.75-year dominant cycle should be a useful planning window for measuring and optimizing capacity, financing, and supply chains.

Consequently, the automobile industry’s average revenue shows a medium-term rhythm but hides innovation-driven firms’ slower and more strategic growth patterns. Thus, the Fourier Transform can distinguish the macro pattern of the entire industry from the business cycle at the firm level.

4.2. EV vs ICE

Tesla, the leading electric vehicle leader, has the same dominant revenue cycle as Toyota and Mercedes-Benz, both of which are ICE-based. This means that Tesla’s long-term expansion has aligned with that of traditional automakers.

However, there’s a significant difference in the Fourier spectrum distribution. Specifically, Tesla's spectrum is characterized by a pronounced concentration around the primary frequency, with minimal energy observed in the high-frequency range (i.e., frequencies greater than 0.2). This finding indicates that Tesla's short-term revenue behavior is characterized by enhanced smoothness

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and predictability. This is likely attributable to the company's direct-to-consumer (DTC) model, real-time integration of production and delivery processes, and centralized global planning.

In contrast, Ford and Volkswagen exhibit shorter dominant cycles (19 quarters) yet also display significantly higher amplitudes in the high-frequency band, suggesting greater short-term volatility. This heightened volatility may be attributed to various factors, including regional sales fluctuations, dealer inventory lags, or seasonal promotional activities. The f = 0.25 frequency, corresponding to annual seasonality (1 cycle per year), is more pronounced for Ford and Volkswagen than for Tesla or Mercedes-Benz, suggesting a more substantial seasonal effect for traditional, combustion-engine-vehicle-heavy, dealer-dependent firms.

Consequently, while the long-term cycles of electric and internal combustion engine car companies may eventually align, their short-term behaviors are significantly divergent. Electric vehicle manufacturers such as Tesla demonstrate excellent stability and reduced sensitivity to seasonal fluctuations, while ICE manufacturers, particularly those operating within a traditional dealer model, exhibit heightened susceptibility to quarterly fluctuations and passive demand cycles.

Dealership VS Direct-to-consumer

Tesla is the only firm among these five that uses a direct-to-consumer strategy. Although it has a long-term cycle, like Toyota and Mercedes-Benz, it has the smoothest spectrum plot. This means that DTC can efficiently reduce revenue noise, probably by eliminating middle-class inventory and reducing delivery delays.

In contrast, those who rely heavily on traditional dealerships show higher amplitude and higher frequency, which means they have irregular short-run shock, probably due to franchise system problems.

As a result, the sales model can affect revenue fluctuation. Direct-to-consumer can lead to lower volatility, less seasonal amplitude, and more predictable financial behaviors.

4.3. Fourier-based ARIMA model

The quarterly revenue model can also be improved.

$$y_t = \mu + \emptyset_1 y_{t-1} + \theta_1 \epsilon_{t-1} + \epsilon_t + a_k cos(\frac{2\pi kt}{T}) + b_k sin(\frac{2\pi kt}{T}) \quad (7)$$

Table 2: Variables for the Fourier-based ARIMA model

Firms	$\emptyset_1$	θ_1	a_k	b_k
Toyota	0.4851	0.2476	2818.003	-425.387
Tesla	0.1425	0.1039	2123.673	1217.716
Ford	0.2539	0.3999	2166.941	1853.803
Volkswagen	0.2539	0.3999	2644.447	1342.371
Mercedes-Benz	0.0634	0.2419	2025.633	1512.989

This is a much more accurate and practical forecasting ARIMA model to predict these auto firms' future revenue.

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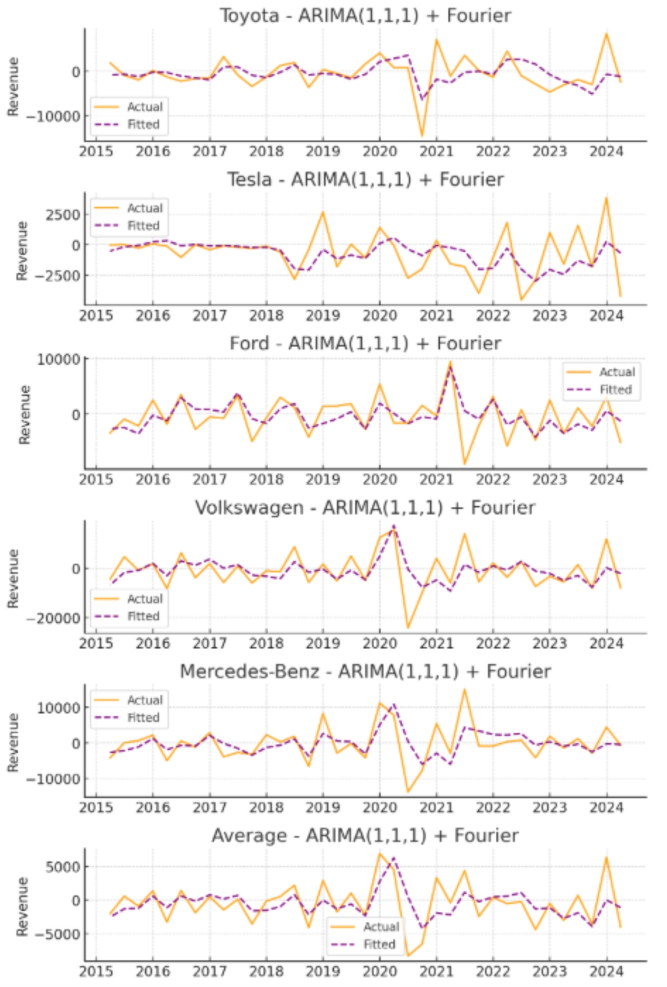

Figure 4: Fitting for the Fourier-based ARIMA model

Table 3: RMSE% for the Fourier-based ARIMA model

Firm	RMSE%
Toyota	4.74
Tesla	9.45
Ford	7.72
Volkswagen	7.72
Mercedes-Benz	8.08

Figure 4 and the RMSE% value show the accuracy of the new ARIMA model.
As a result, the Fourier Transform can help people predict firms' future revenue.

5. Conclusion

This study shows the incredible power of Fourier Transform for analyzing and forecasting quarterly revenue in the automotive industry. By composing revenue's time series data into frequency components, the business cycle can be analyzed further. Toyota, Tesla, and Mercedes-Benz have larger dominant cycles (38 quarters), reflecting stable and innovation-driven revenue patterns, while

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Ford and Volkswagen have smaller dominant cycles (19 quarters), showing sensitivity to the mid-term market.

Additionally, it compares EV vs ICE and dealership vs direct-to-consumer. It shows that ICE-focused firms are more exposed to fluctuations, due to their shorter cyclical pattern. And DTC can reduce revenue noises, making their revenue more predictable.

Finally, the Fourier–ARIMA hybrid model achieves forecasting accuracy with RMSE% values ranging between 4.74% and 9.45%, suggesting it is practical.

In the future, more financial indicators can be implemented, like cash flow, gross margin, and capital expenditure. This can help provide a more comprehensive sight of the industry's economic cycle. What's more, multiple frequency components can be considered. Advanced methods [6] like Wavelet Transform, Empirical Mode Decomposition can then perform better. Focusing solely on revenue provides an incomplete picture of a firm's financial health, as other indicators like cash flow can often be more informative [7]. Additionally, revenue can be distorted by one-time shocks such as COVID-19, which may not reflect the ongoing performance of the business. Moreover, in simplifying complex problems, only one frequency component is considered, meaning that further processing is necessary before applying such analysis in real-world business scenarios.

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