Proceedings of the 2nd International Conference on Computing Innovation and Applied Physics (CONF-CIAP 2023)
DOI: 10.54254/2753-8818/5/20230514

Basic theorems in local class field theory and some further exploration

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Abstract. The relation between the abelian extension of a field and the topological groups of the field itself can be constructed using class field theory. In this piece of writing, the author will introduce the fundamental theorems of local class field theory by conducting a method of literature study. These theorems are the Reciprocity Law and the Existence Theorem, respectively. In addition to that, the author will discuss several unresolved issues in class field theory and provide examples of their applications in number theory. In class field theory, the results can be shown in two different ways. Both can be considered broad strokes. The first step is to demonstrate that the local case is true. By employing the methodology of cohomology and the theory of Lubin and Tate, one can demonstrate the Local Reciprocity Law and the Local Existence Theorem. The fundamental theorems in the global case are going to be demonstrated by utilizing the local results in conjunction with cohomology. Directly demonstrating the Gocal Reciprocity Law is another viable option.

Keywords: local reciprocity law, local existence theorem, group cohomology, Lubin-Tate theory, local class field theory.

1. Introduction

Class field theory is, in essence, a theory about the abelian extension of fields. Hilbert was the first person to establish that there is a bijection between ideal class groups and the abelian extensions of a field. People in the 20th century developed the global class field theory by introducing the idele groups, and they developed the local class field theory by using the language of brauer groups and central simple algebra. Both of these theories were developed in the century.

In this paper, the author will primarily use the method of Lubin-Tate theory and group cohomology to prove the fundamental theorems, and I will demonstrate that the uniqueness of the depth of the local kronecker-weber theorem is equivalent to the depth of the uniqueness of the local artin map.

This proof offers us a means by which we can express the abelian extension in a more explicit manner using the local Kronecker-Weber theorem. Even the local theorem can be used to demonstrate that the global case is correct. In addition to this, we do not even rely on the character of the local field in our proof, which means that we are free to select k as the field of Laurent series.

2. Main theorems

In this section, we shall demonstrate that the Local Reciprocity Law and the Local Existence theorem are correct.

2.1. Local Reciprocity Law

Theorem1: Assume K is a local field, then we can find a unique homomorphism

$$\phi_K: K^{\times} \to \mathrm{Gal}(\mathrm{K}^{\mathrm{ab}}/K)$$

s.t.

(a) Assume π if a prime of K and L/K is a unramified extension of finite degree, then , $\phi_K(\pi)$ restricts on L is $Frob_{L/K}$;

(b) Assume L/K a abelian extension of finite degree, then the norm group $\mathrm{Nm}_{\mathrm{L/K}}(L^{\times})$ belongs to the kernel of $a \mapsto \phi_K(a) \mid L$, and there is an induced isomorphism

$$\phi_{L/K}: \mathrm{K}^{\times}/\mathrm{Nm}_{\mathrm{L/K}}(L^{\times}) \to \mathrm{Gal}(L/K).$$

And the equality that we have here is as follows:

$$\left(K^{\times}:/\mathrm{Nm}_{\mathrm{L/K}}(L^{\times})\right) = [L:K].$$

To begin, we are going to demonstrate the Local Kronecker-Weber theorem, which helps us to describe the abelian closure of a local field K.

Theorem2: For every prime element π of K,

$$\mathrm{K}_{\pi} \cdot \mathrm{K}^{\mathrm{un}} = \mathrm{K}^{\mathrm{ab}}$$

In this thesis, I will use the results in Lubin-Tate theory without proof, for a proof of this results, one can see [1] and [2]

Assume π is a prime of K, then according to Lubin-Tate theory, we can choose a polinomial $f(x) \in F_{\pi}$ and define Λ_n as the root of $f^{(n)}(x)$ and $K[\Lambda_n]$ is a totally ramified extension of degree (q-1)q^n over K with

$$Gal(K[\Lambda_n]) \cong (A/m^n)^{\times}$$

where A is the valuation ring of K, and m is the maximal ideal of A. We function as an inverse limit on both sides, and once we do so, we obtain

$$A^{\times} \cong Gal(K_{\pi}/K)$$

With the above notation, we can define a homomorphism

$$\phi_{\pi}: \mathrm{K}^{\times} \to \mathrm{Gal}\left((K_{\pi} \cdot \mathrm{K}^{\mathrm{un}})/K\right)$$

Let $a \in K^{\times}$. Since we have proved that $K_{\pi} \cap \mathrm{K}^{\mathrm{un}} = \mathrm{K}^{\mathrm{ab}}$, we only need to construct the homomorphism of $\phi_{\pi}(a)$ on K_{π} and $\mathrm{K}^{\mathrm{un}^{\mathrm{un}}}$ separately. Let $a = u\pi^m, u \in U$. We have that $\phi_{\pi}(a)$ restricts on $\mathrm{K}^{\mathrm{un}^{\mathrm{un}}}$ as $Frob^m$, and that it restricts on K_{π} satisfies the following equation.

$$\phi_{\pi}(a)(\lambda) = [u^{-1}]_f(\lambda), \quad \mathrm{all}\lambda \in \cup \Lambda_n.$$

Theorem3: The construction of $K_{\pi} \cdot \mathrm{K}^{\mathrm{un}}$ and ϕ_{π} are independent of π.

Recall that an infinite extension of a complete field is no longer complete, in particular, we can prove K^{un} is not complete. We denote $K^{un,com}$ for its completion, and B for the valuation ring of $K^{\widehat{un,com}}$. σ denotes for the Frobenious automorphism $Frob_K of K^{un}/K$, and so it can be extended to $K^{\widehat{un,com}}$.

Lubin-Tate theory shows that there is a power series: $\theta(X) \in B[[X]]$ satisfying the following properties:

(a) $\theta(T) = \varepsilon T +$ terms of degree ≥ 2;

(b) $\sigma\theta = \theta \circ [u]_f$;

(c) $\theta\left(F_f(X,Y)\right) = F_g\left(\theta(X), \theta(Y)\right)$;

DOI: 10.54254/2753-8818/5/20230514

(d) $\theta \circ [a]_f = [a]_g \circ \theta$
Proof: [1] and [3]
Proof: The independence of $K_\pi \cdot K^{un}$
Let π and $\varpi = \pi u$ be two prime elements of K. Where u is unit of $O_K 0$
then we have

$$(\sigma\theta) \circ [\pi]_f = \theta \circ [u]_f \circ [\pi_f] = \theta \circ [\varpi]_f = [\varpi]_g \circ \theta,$$

Thus

$$(\sigma\theta)(f(T)) = g(\theta(T)).$$

Which means that,$\forall \alpha \in K^{\mathrm{al}}$

$$f(\alpha) = 0 \Rightarrow g(\theta(\alpha)) = 0,$$

$$g(\alpha) = 0 \Rightarrow f(\theta^{-1}(\alpha)) = 0.$$

Thus θ defines a bijection $\Lambda_{f,1} \to \Lambda_{g,1}$

$$K^{un,com}[\Lambda_{g,1}] = K^{un,com}[\theta(\Lambda_{f,1})] \subset K^{un,com}[\Lambda_{f,1}] \subset K^{un,com}[\theta^{-1}(\Lambda_{g,1})] \subset K^{un,com}[\Lambda_{g,1}]$$

and

$$K^{un,com}[\Lambda_{g,1}] = K^{un,com}[\Lambda_{f,1}].$$

$$K^{un,com}[\Lambda_{g,1}] \cap \mathrm{K}^{\mathrm{un}} = \mathrm{K}^{\mathrm{un}}[\Lambda_{g,1}], \quad K^{un,com}[\Lambda_{f,1}] \cap \mathrm{K}^{\mathrm{un}} =^{\mathrm{al}} \mathrm{K}^{\mathrm{un}}[\Lambda_{f,1}],$$

The equality holds because everey subfield E of K^{al} containing K is closed, thus the left side is a cloed field.

$$\mathrm{K}^{\mathrm{un}}[\Lambda_{g,1}] = \mathrm{K}^{\mathrm{un}}[\Lambda_{f,1}].$$

We can use the similar proof to show that

$$\mathrm{K}^{\mathrm{un}}[\Lambda_{g,n}] = \mathrm{K}^{\mathrm{un}}[\Lambda_{f,n}]$$

Holds for all n, thus $\mathrm{K}^{\mathrm{un}} \cdot K_\varpi = \mathrm{K}^{\mathrm{un}} \cdot K_\pi$.
Proof: The independence of ϕ_π
It suffices to prove that the following equation hold for $\forall\pi$ and $\forall\varpi$,

$$\phi_\pi(\varpi) = \phi_\varpi(\varpi).$$

From the above equation we have that for $\forall\pi'$

$$\phi_{\pi'}(\varpi) = \phi_\varpi(\varpi) = \phi_\pi(\varpi).$$

Which means $\phi_\pi = \phi_{\pi'}$ hold on $K^\times$
To show the equality, we only need to show that $\phi_\pi(\varpi)$ and $\phi_\varpi(\varpi)$ agree on both K^{un} and K_ϖ
For K^{un}, $\phi_\pi(\varpi)$ and $\phi_\varpi(\varpi)$ equals to Frobenius automorphism.
Now we prove they agree on K_ϖ.
From theorem3 we know that there is an isomorphism $\theta: F_f \to F_g$ over $K^{un,com}$, and we have an induced isomorphism of the roots $\Lambda_{f,n} \to \Lambda_{g,n}$ for all n. Then, notice that $\phi_\varpi(\varpi)$ is the identity on K_ϖ, $K_{\varpi,n}$ is generated over K by the image of roots $\theta(\lambda)$ for $\lambda \in \Lambda_{f,n}$, it suffices to prove that

$$\phi_\pi(\varpi)(\theta(\lambda)) = \theta(\lambda), \quad \text{all } \lambda \in \Lambda_{f,n}.$$

denotes $\varpi = u\pi$. Then $\phi_\pi(\varpi) = \phi_\pi(u) \cdot \phi_\pi(\pi) = \tau\sigma$, thus

DOI: 10.54254/2753-8818/5/20230514

$$\sigma = \begin{cases} \text{Forb}_K \text{ on } K^{\text{un}} \\ \text{id} \quad \text{on } \lambda \end{cases} \quad \tau = \begin{cases} \text{id} \quad \text{on } K^{\text{un}} \\ [u^{-1}]_f \quad \text{on } \lambda \end{cases}$$

Since from the construction we know that the series θ has coefficients in $\mathrm{K}^{\text{un,com}^{\text{un}}}$

$$\phi_\pi(\varpi)\big(\theta(\lambda)\big) = \tau\sigma\big(\theta(\lambda)\big) = (\sigma\theta)(\tau\lambda) = (\sigma\theta)\big([u^{-1}]_f(\lambda) = \theta(\lambda).$$

From which we obtain the independence.

According to the constrution of $K_{[\pi,n]}$, we can draw the equality that

$$Nm\big(K_{[\pi,n]}\big) = (1+m^n)\cdot\pi^Z$$

which means that π is a norm of $K_{[\pi,n]}$ for all n.

From the above statements, we have constucted a homomorphism

$$\phi_\pi: K^\times \to \text{Gal}\big((K_\pi \cdot \mathrm{K}^{\text{un}})/K\big)$$

such that

$*$ 1 $\phi_\pi(\pi) \mid \mathrm{K}^{\text{un}^{\text{u}}} = \text{Frob}_\text{K}$;

$*$ 2 for all m and n, $\phi_\pi(a) \mid \big(K_{\pi,n}\cdot K_m\big)$ = id for $a \in (1+\mathfrak{m}^n)\cdot\langle\pi^m\rangle$.

$*$ 1 come from the definition directly, and $*$ 2 holds because

$$Gal(K[\Lambda_n]) \cong (A/m^n)^\times$$

Thus the kernel of $\phi_\pi(\cdot) \mid \big(K_{\pi,n}\cdot K_m\big)$ is $(1+m^n)\cdot\langle\pi^m\rangle$.

What's more, we have shown that the construction of $K_\pi \cdot K^{\text{un}}$ and ϕ_π don't rely on the choice of π.

We shall now use some cohomoloy tools to prove the existence of the homomorphism described in the Local Reciprocity Law.

The following theorem would be very important to the solution:

Theorem: G is a finite group(not necessary abelian) and C is a $G-$module with the following properties. For all subgroups H of G,

(a) $\mathrm{H}^{1^1}(H,C) = 0$

(b) $\mathrm{H}^{1^2}(H,C) \cong \mathrm{Z}/(H:1)\mathrm{Z}$.

Then, there is an isomorphism between two Tate cohomology groups

$$H_T^r(G,\mathbb{Z}) \to H_T^{r+2}(G,C)$$

Proof: [1] and [4].

Since $(G, L^\times)$, where G is a finite Galois extension of local field L/K satisfies the conditions of Tate's theorem. Thus we obtain a homomorphism.

L/K is a finite Galois extension, then the homomorphism

$$H_T^r(\mathrm{l}(L/K),\mathbb{Z}) \to H_T^{r+2}(\mathrm{l}(L/K),L^\times)$$

Induced by the map $x \mapsto x \cup u_{L/K}$ is an isomorphism. If we let $r = -2$, then we can obtain the isomorphism for the abelian quotient of G

$$G^{\text{ab}} \simeq K^\times/\text{Nm}_{\text{L/K}}(L^\times)$$

By using cohomology tools, we can explicitly describe the inverse map through

$$\phi_{L/K}: K^\times/\text{Nm}_{\text{L/K}}(L^\times) \to \text{Gal}(L/K)^{\text{ab}}$$

And we could not have found a better local map of Artin than this one.

DOI: 10.54254/2753-8818/5/20230514

In order to demonstrate that the Local Reciprocity Law is true, we are going to employ the theorems that have been presented thus far, focusing primarily on the building of $K_{\pi,n}$ and $\phi_\pi(\pi)$, as well as the development of the Local Artin Map.

Lemma: For all $a \in K^\times, \phi(a) \mid K_\pi \cdot K^{un} = \phi_\pi(a)$.

Proof: ∀ prime π of $K, \phi(\pi)$ becomes the identity map on $K_{\pi,n}$ since π is a norm of $K_{\pi,n}$, and $\phi_\pi(\pi)$ (which is constructed in the local Kronecker-Weber theorem, and is independent of π) acts trivially on $K_{\pi,n}$. Since $\phi(\pi)$ and $\phi_\pi(\pi)$ both act as $Frob_K$ on K^{un}. Thus they are the same on $K_\pi \cdot K^{un}$. And since $K^\times$ is generated by the primes of K as a group($a \in K^\times$can be written $a = u\pi^r$, and $u = (u\pi)\pi^{-1}$), the equality comes.

To show the uniqueness, It remains to prove that

$$K_\pi \cdot K^{un^{\mathrm{un}}} = K^{ab^{\mathrm{ab}}}$$

Proof: Let

$$K_{n,m} = K_{\pi,n} \cdot K_m,$$

and

$$U_{n,m} = (1 + \mathfrak{m}^n) \cdot \langle \pi^m \rangle.$$

We are given that $\phi_\pi(a) \mid K_{n,m} = 1$ for all $a \in U_{n,m}$. Hence $\phi(a) \mid K_{m,n} = 1$ for all $a \in U_{n,m}$, and so $U_{n,m} \subset \mathrm{Nm}(K_{n,m}^\times)$. But

$$\begin{aligned} (K^\times : U_{n,m}) &= (U : 1 + \mathfrak{m}^n)(\langle \pi \rangle : \langle \pi^m \rangle) \\ &= (q-1)q^{n-1} \cdot m \\ &= [K_{\pi,n} : K][K_m : K] \\ &= [K_{m,n} : K], \end{aligned}$$

ϕ induces an isomorphism

$$K^\times / \mathrm{Nm}(K_{n,m}^\times) \to \mathrm{Gal}(K_{n,m}/K).$$

Thus,

$$U_{n,m} = \mathrm{Nm}(K_{n,m}^\times).$$

L/K is a finite abelian extension. Considering that ϕ specifies an isomorphism starting from $K^\times/\mathrm{Nm}(L^\times)$ to $\mathrm{Gal}(L/K)$, it follows that $\mathrm{Nm}(L^\times)$ has a limited index in $K^\times$. And we can easily verify that the norm group of finite index is open(by considering the compactness, see [5]) , which contains $U_{n,m}$ for some $n, m \geq 0$. Then The map

$$\phi: K^\times \to \mathrm{Gal}(L \cdot K_{n,m}/K)$$

is onto and, for $a \in K^\times$,

$\phi(a)$ fixes the elements of $L \Leftrightarrow a \in \mathrm{Nm}(L^\times)$

$\phi(a)$ fixes the elements of $K_{n,m} \Leftrightarrow a \in \mathrm{Nm}(K_{n,m}^\times) = U_{n,m}$

Because $\mathrm{Nm}(L^\times) \supset U_{n,m}$, this implies that $L \subset K_{n,m}$. It follows that

$K^{ab^{\mathrm{ab}}} = K_\pi \cdot K^{un^{\mathrm{un}}}$.

2.2. Local Existence Theorem

There is a relationship, known as a bijection, that can be established between the open subgroups of fixed index in $K^\times$ and the norm groups of finite abelian extensions in $K^\times$.

Before we prove this theorem, we need some results induced by Local Reciprocity Law, for a detailed information, see the reference [6].

Theorem: K is local field. L/K is a finite abelian extension, then the following properties hold:

(a) $L \mapsto \mathrm{Nm}(L^{\times})$ can be thought of as a bijection between finite abelian extensions of and the norm groups that exist in $K^{\times}$.

(b) $L \subset L' \Leftrightarrow \mathrm{Nm}(L^{\times}) \supset \mathrm{Nm}(L'^{\times})$.

(c) $\mathrm{Nm}((L \cdot L')^{\times}) = \mathrm{Nm}(L^{\times}) \cap \mathrm{Nm}(L'^{\times})$.

(d) $\mathrm{Nm}((L \cap L)^{\times}) = \mathrm{Nm}(L^{\times}) \cdot \mathrm{Nm}(L'^{\times})$.

(e) any subgroup of $K^{\times}$ that contains a norm group is again a norm group.

Proof: one arrow of (b) is trivial since $N_k^{L'} = N_k^{L}\left(N_L^{L'}\right)$

hence for the statement (c) $N(L \cdot L)^{\times} \subset N(L^{\times}) \cap N(L'^{\times})$ conversely, if $a \in N(L^{\times}) \cap N(L'^{\times})$ then $\phi_L(a) = \phi_{L'}(a) = 1 \quad \Rightarrow \phi_{LL'}(a) \Rightarrow a \in N(L \cdot L)^{\times}$

For converse of (b), if $N(L'^{\times}) \subseteq N(L^{\times})$ then $N((L \cdot L')^{\times}) = N(L'^{\times})$ since $[L' \colon K] = |K^{x}/N(L')| = |K^{\times}/N(LL')^{\times}| = [(L \cdot L') \colon K]$ thus $L \subseteq L^{\times}$

For (a), the bijection comes from the fact that it is surjective, and for injective, it follows from (b).

As for (e), consider $N = N(L^{\times})$ to be the norm group of the abelian extension L/K, and $N \subseteq I$ then $\phi_L(I) \mapsto \mathrm{Gal}(L/K)$ maps I into a subgroup of Gal(L/K), consider it as H, and the fixed field L^H as L' then $\mathrm{Gal}(L' \mid K) \subseteq Gal(L|K)$ and the kernel of $\phi_{L'}$ is $N(L'^{\times})$. On the other hand, the kernel is the elements in $k^{\times}$ such that $\phi_L(a) \subseteq \phi_L(I)$, which is I, Thus $I = N(L)$

For (d). Since the largest subextension of L contained in L' is L ∩ L′, and the smallest subgroup of $\mathrm{Nm}(L^{\times})$ containing $\mathrm{Nm}(L'^{\times})$ is $\mathrm{Nm}(L^{\times}) \cdot \mathrm{Nm}(L'^{\times})$, which is a norm group since it contains a norm group, thus according to (a),

The equality holds.

Proof for the Local Existence Theorem: we have already proved the injection by the above lemma, as for surjection. Every open subgroup of bounded index is a norm group if it contains a norm group, which follows from the fact that every such group includes $U_{n,m}$. Then we've proven Local existence.

3. Further exploration with the help of cohomology

In this part, the author will present some propositions about the local artin map. Since in the above proof, we just simply constructed the local Artin map by the Local Kronecker-Weber theorem, and we do not have an explicit description on a arbitrary field. With the help of cohomology, we can obtain various results of norm groups.

L/K is a finite Galois extension, denotes $G = \mathrm{Gal}(L/K)$. Then both L (consider L as a addictive group) and $L^{\times}$(consider it as a multiplicative group) are G-modules.

Theorem: L/K is a finite Galois extension whose Galois group is G. Then $H^1(G, L^{\times}) = 0$

Proof: Let $\varphi \colon G \to L^{\times}$be a crossed homomorphism, which means

$$\varphi(\sigma\tau) = \sigma\varphi(\tau) \cdot \varphi(\sigma), \quad \sigma, \tau \in G,$$

and it suffices to find a $c \in L^{\times}\ s.t. \varphi(\sigma) = \sigma c/c$. For $a \in L^{\times}$, let

$$b = \sum_{\sigma \in G} \varphi\,(\sigma) \cdot \sigma a.$$

Suppose $b \neq 0$. Then

$$\tau b = \sum_{\sigma} \tau\, \varphi(\sigma) \cdot \tau\sigma a = \sum_{\sigma} \varphi\,(\tau)^{-1}\varphi(\tau\sigma)\tau\sigma a = \varphi(\tau)^{-1} b$$

Hence

$$\varphi(\tau) = b/\tau b = \tau(b^{-1})/b^{-1},$$

Thus φ is principal. We only need to prove the existence of a s.t. $b \neq 0$. L is a field and H is a group; then every finite set $\{f_i\}$ of distinct homomorphisms $H \to L^{\times}$is linearly independent over L, i.e.,

$$\sum a_i f_i(\alpha) = 0 \quad \text{all}\alpha \in H \Rightarrow a_1 = a_2 = \cdots = a_n = 0.$$

Thus, $\sum_\sigma \varphi(\sigma)\sigma \neq 0$, from which we obtain the existence of a s.t. $b \neq 0$.

From the above theorem, we can easily obtain the following,

Theorem: L/K is a cyclic extension, and σ is the generator of the Galois group. If $Nm_{L/K}a = 1$, then $a=\frac{\sigma b}{b}$ for some b $\neq 0$.

Theorem: L/K is a finite Galois extension. Then $H^r(G, L)$=0 for all $r > 0$.

Proof: According to Normal Basis Theorem, $\exists\alpha \in L$ such that $\{\sigma\alpha \mid \sigma \in G\}$ provides supports for L as a K-vector space. Then $(\sigma\alpha)_{\sigma\in G}$ defines an isomorphism of G-modules

$$\sum_{\sigma\in G} a_\sigma \sigma \mapsto \sum_{\sigma\in G} a_\sigma \sigma\alpha : K[G] \to L.$$

But $K[G] = \mathrm{Ind}_{\{1\}}^{G} K$, and so $H^r(G, L) \simeq H^r(\{1\}, K) = 0$ for $r > 0$

The results that were presented earlier are commonly known as Hilbert's Theorem 90. And for another proof of this, see [7].

Theorem: L is a finite extension of K, then make E turn to be the largest abelian subextension L/K; then

$$\mathrm{Nm}_{\mathrm{L/K}}(L^\times) = \mathrm{Nm}_{\mathrm{E/K}}(E^\times).$$

Proof: We only prove the local case, for a global proof, see [8].

Since L contains E, then $\mathrm{Nm}_{\mathrm{L/K}}(L^\times)$is contained in $\mathrm{Nm}_{\mathrm{E/K}}(E^\times)$. The proof for Galois case is trivial, and we only consider the general case

Denotes L'/K to be a Galois extension which contains L. Let $G = \mathrm{Gal}(L'/K)$ and $H = \mathrm{Gal}(L'/L)$. Since the largest ablian extension of K in L is E, then $G' \cdot H$ is the fixed group,($G' = \{aba^{-1}b^{-1} \mid a, b \in G\}$), Consider The $\phi_{L'/K}(a) \in G/G'$ maps to 1 in $G/G'H$. since $\phi_{L'/L}$ is surjective, then $\exists b \in L^\times, \mathrm{S.t.}\ \phi_{L'/K}(a) = \phi_{L'/K}(\mathrm{Nm}(b))$, thus $\exists \mathrm{c} \in \mathrm{L}'^\times, a/\mathrm{Nm}(b) \in \mathrm{Nm}(\mathrm{c})$

Therefore, we have

$$a = \mathrm{Nm}_{\mathrm{L/K}}\left(b \cdot \mathrm{Nm}_{L'/K}(c)\right) \in \mathrm{Nm}_{\mathrm{L/K}}(L^\times)$$

With this theorem we see that the norm group of a local field cannot be used to categorize its nonabelian extension.

4. Conclusion

In this study, the author uses Lubin-Tate theory and cohomology theory to show some of the fundamental theorems of local class field theory. The Galois group of abelian extensions has, in fact, been described in greater depth. The cohomology group of K can be constructed using brauer groups and characterized using a central simple algebra. Galois representation and Galois cohomology are just two examples of where the conclusions of class field theory can be put to use.

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DOI: 10.54254/2753-8818/5/20230514

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