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Determinism in hidden variable interpretation and Turing's halting problem

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Abstract. Heisenberg deduced the famous uncertainty principle which shows that there exist several conjugated quantities that can never be measured precisely at the same time. Further, Copenhagen interpretation gives a new perspective about quantum mechanics, which claims that particles do not have properties like position or momentum until people measure it. Hence, the famous EPR paradox was proposed to question the realism and locality of Quantum mechanics, which leads to the Hidden Variable explanation. However, this theorem was proved wrong by John Bell in 1964 with Bell Inequality. In addition, Hidden Variable Interpretation was developed by De Broglie and Bohm, they came up with the Bohmian Mechanics, which can be considered as Non-local Hidden Variable theorem. This interpretation gives physical meaning to waves. Unlike Copenhagen Interpretation, this theorem claims that particles do have a determined position. Some people may argue that it can be proved contradicted by using the Turing method (self-reference). Because if the algorithm represented by physics law described Hidden Mechanics to determine particles' state, then the future is determined and predictable. Therefore, another algorithm can be established based on that, which will lead to "Liar Paradox". This article will briefly introduce the uncertainty principle and some interpretations about quantum mechanics. Furthermore, this article will combine some ideas in Alan Turing's Halting Problem to the universe of non-local hidden variables as a thought experiment, which involve self-reference, to give an interesting result of locality and determinism.

Keywords: determinism, liar paradox, discussion of determinism in quantum mechanics.

1. Introduction

Idea of determinism was originated by Stoicism in their universal causality theory, which became mature as Newtonian Mechanics had developed. Basically, if the position of a particle is known, and it is interacted with another particle with known velocity, then the trace of them can be predicted by Newtonian Mechanics. As the book The Concept of Physical Law written by Swartz Norman denoted, Determinism gives us a picture of a clockwork universe, which does not reserve a place for free will. The famous thought experiment of Determinism is Laplace Demon, which was proposed by Laplace in 1840[1].

However, the universe picture Quantum Mechanics established refutes Determinism and Realism. Heisenberg first gave the uncertainty relation between position and momentum in Physical Principles of Quantum Theory, which denoted δx and δp should in the form:

$$\delta x \delta p \gtrsim h \tag{1}$$

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And in 1927, Earl Kennard proved the modern inequality:

$$\delta x \delta p \geq \frac{\hbar}{2} \tag{2}$$

These equations denote that the position and momentum cannot be measured at the same time, if people get more precise to position, then they will get larger uncertainty of momentum. As for the reason, one of the most obvious explanation is the mathematical model people used, which can be written as multiple superposition of e^{ikx}, representing the probability density of a particle in the universe. As more $e^{ik_n x}$ added together (δk gets bigger), the more wave package will be narrow (δxgets smaller). Surely, most experiment results fit with that conclusion. However, it is still worth asking if the uncertainty principle is the "flaw" of the wave function, or if it is the basic rule of the universe. Is determinism truly invalid?

The Copenhagen Group (basically attributed to Niels Bohr and Werner Heisenberg) gives the Copenhagen Interpretation, which claims that particles do not have portion and momentum until measurements occur. However, many physicists did not like this explanation, so Einstein, Podolsky and Rosen gave a local Hidden Variable Interpretation, which preserved the determinism but was proved wrong by Bell Inequality. However, Hidden Variable Theory was developed by De Broglie and Bohm, denoted as Bohmian mechanics (non-local hidden variable). Also, there are other interesting interpretations like Many-World-Interpretation. However, these two interpretations mentioned are untestable, because they give the same result as the Copenhagen Interpretation.

However, it seems that the idea that future is determined and predictable can always be violated. In 1936, Alan Turing posted his famous paper On Computable Numbers With an Application to the Entscheidungsproblem, which proved there does not exist a general algorithm to determine if a Turing machine would halt or not. Basically, if people use a similar method as Alan Turing, assume that determinism is true, and build a setup to represent the "Liar Paradox" in the physical universe. A thought experiment can be established by discussing a prediction machine and an observer always give the antithetical result to that machine.

In the main body of this paper, this view can be illustrated by introducing the statement of Copenhagen interpretation and the sketch of Turing proof, then combining them to see whether determinism (hidden variables) is effective.

2. Main body

2.1. Heisenberg Uncertainty Principle and Copenhagen Interpretation

The first version of the Uncertainty Principle that Heisenberg gave is in The Physical Principles of the Quantum Theory in 1927. In Chapter 2, Heisenberg denoted that:

"the statement that the position of an electron is known to within a certain accuracy Δx at the time t can be visualized by the picture of a wave package in the proper position with an approximate extension Δx."[2]

Basically, if people have a wave vector k_0, and write a plane as following,

$cos(k_0 - \frac{\Delta k}{2})x + cos(k_0 + \frac{\Delta k}{2})x$, where $\Delta k \ll k_0$.

They can simply reform it to,

$$2cos(k_0 x)cos(\frac{\Delta k}{2} x) \tag{3}$$

As for the length wave package which denoted as Δx, they follow the relation:

$\Delta x \Delta k \approx 2\pi.$

However, if they insert multiple plane wave in to it,

$cos(k_1 x) + cos(k_2 x) + cos(k_3 x) + \ldots + cos(k_n x)$

where n is discrete integer and finite, and

$k - \frac{\Delta k}{2} \leq k_1, k_2 \ldots k_n \leq k + \frac{\Delta k}{2}$

Then, they can get the relation of $\Delta x, \Delta k$:

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$\Delta x \Delta k \gtrsim 2\pi$,
where Δk can be replaced by Δp,

$$\Delta x \Delta p \gtrsim h \tag{4}$$

The rough relation of Δx and Δk can be directly and virtually derived from the sum of several plane waves. Moreover, people can modify it with Fourier Decomposition to make the result become more precise. For example, in the book Quantum Mechanics, Volume I-John Wiley & Sons (1977) , Claude Cohen-Tannoudji, Bernard Diu, Franck Laloë denoted that, Gaussian Function can maximumly reduce the uncertainty, so people can use plane waves to composite a Gaussian Function[3]. Set $g(k) = e^{\frac{-(k-k_0)^2}{2\Delta k^2}}$,

$$\psi(x,0) = \int_{-\infty}^{\infty} e^{\frac{-(k-k_0)^2}{2\Delta k^2}} e^{ikx} dk$$

$$= e^{ik_0 x} \int_{-\infty}^{\infty} e^{\frac{-\kappa^2}{2\Delta k^2} + i\kappa x} d\kappa, where \kappa = k - k_0$$

$$= e^{ik_0 x} e^{\frac{-\Delta k^2 x^2}{2}} \int_{-\infty}^{\infty} e^{\frac{-1}{2\Delta k^2}(\kappa - i\Delta k^2 x)^2} d\kappa,$$

$$= \sqrt{2\Delta k^2 \pi} e^{ik_0 x} e^{\frac{-x^2 \Delta k^2}{2}} \tag{5}$$

Notice that it is also a Gaussian Function of x with uncertainty $\frac{1}{\Delta k}$. Hence, it gives us that, $\Delta x \Delta k \geq 1$ which is,

$$\Delta x \Delta p \geq \hbar.\mathrm{s} \tag{6}$$

Although the Copenhagen Interpretation has no unique definition, Bohr and Heisenberg had a very different interpretation to the phenomena in Quantum Mechanics. However, it can still be concluded by some common statement shared by them. According to John Faye, in his paper Copenhagen Interpretation of Quantum Mechanics that published in 2003, Faye attributed a few claims as following[4]:

Quantum Mechanics is in-deterministic. Basically, Copenhagen Interpretation claimed that determinism is invalid, particles do not have position or momentum until they are measured. This statement is illustrated more detailed in the Complementary rule below.

Complementary: in the same system, some property cannot be discussed at the same time. The condition of the measuring experiment should be considered, when it comes to specific properties. And there are some exclusive quantities that cannot be measured together, like position and momentum. As Faye concluded:

"…the state of the measuring device and the state of the object cannot be separated from each other during a measurement but they form a dynamical whole. Bohr called this form of holism "the individuality" of the atomic process. Thereby, he had in mind not only that the interaction is uncontrollable but also that the system-cum-measurement forms an inseparable unity due to the entanglement…" [4]

The Corresponding Rule: Quantum Mechanics will represent the same result as Classical Mechanics in a large quantum number (large obit or energy). For example, according to Ehrenfest's Theorem, Expectation value of momentum and position will satisfy classical equations of motion at a very large potential scale.

However, the statement that certain quantities should be defined despite the laboratory context, and the claim that quantities do not have meanings before measuring in Copenhagen Interpretation, caused dissatisfaction among many physicists. Physicists represented by Einstein believed that there should be a deterministic interpretation of physical phenomena and Quantum mechanics should also follow the requirement of realism. Hence, Einstein gives the local hidden variable interpretation, which claims that the quantum quantities are determined locally by some unknown mechanics people do not know yet.

However, it was proved wrong by John Bell in 1964 using Bell Inequality[5]. De Broglie and Bohm developed the hidden variable, and they gave a non-local hidden variable interpretation which is called Bohmian Mechanics.

2.2. Non-local Hidden Variable Interpretation: Bohmian Mechanics

Bohmian Mechanics claims that the wave function and particle are all real and have their own physical meaning. Basically, the particles are guided by a pilot wave that can be described by wave function. The velocity of a single particle can be described by the following equation call guiding equation:

$$\frac{d\vec{Q}_k}{dt}(t) = \frac{\hbar}{m} Im\left(\frac{\nabla\psi}{\psi}\right)(\vec{Q}_1, \vec{Q}_2, \ldots, \vec{Q}_n, t)\,[6] \tag{7}$$

And the guiding wave also is governed by the time involved Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}\psi = -\sum_{k=1}^{n}\frac{\hbar}{2m_k}\nabla_k^2\psi + V\psi\ [6] \tag{8}$$

Therefore, the hidden variable in this theorem is the initial position of particles. In addition, this theorem gave the same result as the Copenhagen Interpretation, so it is untestable.

Bohmian Mechanics used nonlocal hidden variables to avoid the collapse of wave function, and it gives physical meaning to the wave function as pilot wave. Also, it gives a deterministic interpretation to the quantum phenomena, which denoted that particles have specific position and momentum before measuring, and the reason that uncertainty principle occurred is due to lack of knowledge of the particles' initial position.

Bohemian Mechanics gives the same experiment result as Copenhagen Interpretation, because although it gives physical meaning to the wave function as pilot wave, this pilot wave strictly follows the Schrödinger equation and guides motions of particles. However, Bohmian Mechanics become more nonlocal than Copenhagen Interpretation when it comes to multi-particles spread in large distance in the universe. From the equations above, it is noticed that every particle's velocity influences others despite the distance between them. And this wave can be suddenly changed when a particle's state changes. This is one of the most essential criticisms of Bohmian Mechanics from the mainstream.

In 1936, Alan Turing posted his famous paper On Computable Numbers With an Application to the Entscheidungsproblem, in which he proved there does not exist a general algorithm to solve the halting problem. In other words, whether a certain algorithm will halt or not is undecidable until people actually execute it.

Eric C. R. Hehner used Pascal to show how Alan proved that statement. According to Hehner, people can assume there is an algorithm 'halts (p, i)', which returns True if p represents a Pascal procedure terminated with the given input i, and returns False otherwise. Then, it can always write another procedure based on 'halts (p, i)', which is called 'twist(s)'. The procedure 'twist(s)' terminates when 'halts (p, i)' gives True, otherwise it gives the result itself (means not halts).

Basically, the modern Halting Problem can be programed in **Figure 1**[7]:

```
function halts(p, i: string) : boolean;
{return true if p excecution terminates with given input parameter i}
{return false otherwise}
procedure twist(s: string);
begin
    if halts(s, s) then twist(s)
end
```

Figure 1. Modern Halting Problem programming.

Hehner denoted that for any oracle algorithm 'halts(p, i)' which can predict any other algorithm will halt to not, if twist(twist) terminate, then 'halts(twist, twist)' returns true, and from the body of the program, the twist(twist) will not terminate. And if twist(twist) does not terminate, then 'halts(twist, twist)' returns false, the twist(twist) will terminate. This is a program version of 'Liar Paradox', which shows that the general algorithm that solves the Halting Problem does not exist.

In conclusion, Halting Problem using contradiction shows that any Oracle algorithm will have paradox when it comes to self-reference. It is always possible to build another algorithm based on that which leads to a contradiction and makes the prediction wrong. Nevertheless, it seems that if a system is determined, the possible Oracle algorithm mentioned above seems valid. Because a determined system can always be described and predicted using the corresponding physics law (the hidden variable mechanics) mathematically. Therefore, is it possible to use a similar method to show the determinism and hidden variable theorem is a 'Liar Paradox' which is always logically invalid.

2.3. Turing's *Method* in a Deterministic Universe

Firstly, it will be necessary to conclude the attributes of local and nonlocal Hidden Variable Interpretation. Either the local or nonlocal hidden variable theorem claims that the state of the particle is determined by some hidden mechanics that people do not know yet, and the position and momentum of particles are determined.

Hence, people may argue that whether there exists an algorithm $h(t, \vec{q}_1, \vec{q}_2, \dots \vec{p}_1, \vec{p}_2, \dots)$ which can be represented in a mathematical way. For complement, $h(t, \vec{q}_1, \vec{q}_2, \dots \vec{p}_1, \vec{p}_2, \dots)$ is a group of algorithm to describe the physics law of hidden mechanics that determines particles' states, and $\vec{q}_i$, $\vec{p}_i$ is the position and momentum of a particle. For simplicity, write $h(t, \vec{q}_1, \vec{q}_2, \dots \vec{p}_1, \vec{p}_2, \dots)$ as $h(t, \vec{Q}, \vec{P})$, where $\vec{Q}$, $\vec{P}$ are $[\vec{q}_1\, \vec{q}_2 \dots \vec{q}_n]$ and $[\vec{p}_1\, \vec{p}_2 \cdots \vec{p}_n]$.

In this case, people can use the method above to write another algorithm $g(s)$ based on $h(t, \vec{Q}, \vec{P})$. And because $g(s)$ and input parameter s can be established in physics world, assume g contains particles: $(\vec{q}_j, \vec{p}_j)$, $(\vec{q}_{j+1}, \vec{p}_{j+1})$, …, $(\vec{q}_{k-1}, \vec{p}_{k-1})$, $(\vec{q}_k, \vec{p}_k)$. Then, people could rewrite $h(t, \vec{Q}, \vec{P})$ as $h(t, g, \vec{Q}', \vec{P}')$.

$h(t, \vec{Q}, \vec{P})$ can determine if a certain algorithm f will terminate in a time duration Δt, because basically what $h(t, \vec{Q}, \vec{P})$ does is running f in advance before in the real world. Hence, people can do the similar things as the previous section as shown in **Figure 2**:

function $h(t + \Delta t, f, \overrightarrow{Q'}, \overrightarrow{P'})$: **boolean**;
{return $true\ if\ f$ excecution terminatesfrom t to $t + \Delta t$,}
{return $false$ otherwise}
procedure g;
begin
  if $h(t + \Delta t, g, \overrightarrow{Q'}, \overrightarrow{P'})$ **then** g
end

Figure 2. Modern Halting Problem programming 2.

This seems good, because it successfully leads to the contradiction. However, the procedure above has a slight difference compared to the previous section. Notice the parameter of g is missing. Because the input in $h(t, \vec{Q}, \vec{P})$ are the information of real world particles, there can never exist two same input parameter s in h (which become $h(t, s, s, \vec{Q}', \vec{P}')$). Therefore, it is impossible to have $g(g)$ which leads to the contradiction. And Hehner denoted this version above as the "intermediate" version:

"This is still not a programming problem, not a computability problem, not a lack of expressiveness of Pascal. It is still the same inconsistency that was present in the unparameterized, unencoded version, and the same inconsistency that was present in the twist procedure."

The main issue is that, Turing method is dealing with an unknown function $h(x)$ where the same parameter is valid. However, in our version, how the $h(x)$ work has already been defined, which is calculating a certain event in advance in the real world in order to predict the future. Therefore, it can be re-considered more physically by establishing a thought experiment.

Assuming $h(t, \vec{Q}, \vec{P})$ as a black box with an output signal light as shown in **Figure 3**, the only thing people know about this black box is using the algorithm $h(t, \vec{Q}, \vec{P})$ and need the input $\vec{Q}_{initial}$ and $\vec{P}_{initial}$.

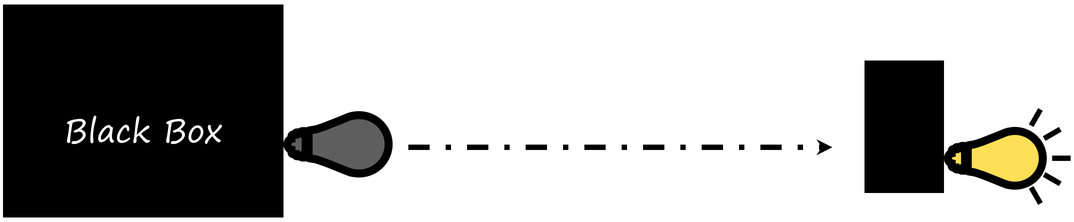

Figure 3. Physical Simulation of Halts Problem.

Then, $g(s)$ can be defined as a detector with a signal light. When the detector detects that signal light of $h(t, \vec{Q}, \vec{P})$ is off, the signal light of $g(s)$ will be on. And if the detector detects the signal light of $h(t, \vec{Q}, \vec{P})$ is on, the signal light of $g(s)$ will be off. After setting the $g(s)$, let $h(t, \vec{Q}, \vec{P})$ to predict the state of signal light on $g(s)$ after particular time duration Δt, and the output of $h(t, \vec{Q}, \vec{P})$ is the signal light on $h(t, \vec{Q}, \vec{P})$, if $h(t, \vec{Q}, \vec{P})$ find that the light $g(s)$ will be on, then light on $h(t, \vec{Q}, \vec{P})$ will be on, vise versa, and when the $h(t, \vec{Q}, \vec{P})$ executes, the light on $h(t, \vec{Q}, \vec{P})$ is always off.

In this case, $h(t, \vec{Q}, \vec{P})$ can never predict the correct future. Theoretically, the algorithm $h(t, \vec{Q}, \vec{P})$ should give the correct answer of all particles in the universe all the time.

Some may argue this setup is tricky, because $h(t, \vec{Q}, \vec{P})$ can never give correct answer in this situation, and $h(t, \vec{Q}, \vec{P})$ may execute forever. In other word, $h(t, \vec{Q}, \vec{P})$ will refuse to give the prediction, which means it will never terminate. Hence, it will be necessary to discuss three situations separately.

The first situation is the light on $h(t, \vec{Q}, \vec{P})$ is on, and $h(t, \vec{Q}, \vec{P})$ gives the false prediction.

The second and third case is the light on $h(t, \vec{Q}, \vec{P})$ is off, one possible case is $h(t, \vec{Q}, \vec{P})$ gives the false prediction, and the other one is $h(t, \vec{Q}, \vec{P})$ will not terminate, which means $h(t, \vec{Q}, \vec{P})$ is not able to predict $g(s)$.

Conclusively, the case that light on $h(t, \vec{Q}, \vec{P})$ is on means $h(t, \vec{Q}, \vec{P})$ gives the wrong prediction, and the case that the light on $h(t, \vec{Q}, \vec{P})$ is off means that $h(t, \vec{Q}, \vec{P})$ gives the wrong prediction or $h(t, \vec{Q}, \vec{P})$ is not able to give the prediction on $g(s)$.

However, it is not possible to use $h(t, \vec{Q}, \vec{P})$ to predict something else except things that self-related to $h(t, \vec{Q}, \vec{P})$? Not likely, because in the example above, $g(s)$ used a signal light as output. But the signal light can be replaced by an event that has a massive impact on the environment. In that case, all the things that influenced by that event will become unpredictable, or $h(t, \vec{Q}, \vec{P})$ will give the incorrect prediction. Therefore, if the place where information of the signal from $g(s)$ can be detected, $h(t, \vec{Q}, \vec{P})$ cannot give the prediction of that position.

2.4. More About This Thought Experiment

The thought experiment above, which combine Turing's method with determinism, gives the result that $h(t, \vec{Q}, \vec{P})$ cannot give the prediction. For example, in **Figure 4**, at time t_0 in real universe, $h(t, \vec{Q}, \vec{P})$

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was inputted a group of data $(\vec{Q}_{t_0}, \vec{P}_{t_0})$, in which $(\vec{Q}_{t_0}, \vec{P}_{t_0})$ represent the universal particles' information at t_0, in order to calculate $(\vec{Q}_{t_1}, \vec{P}_{t_1})$. If $h(t, \vec{Q}, \vec{P})$ would not terminate even though the time in real world had pass though t_1, then what will happen if we input same group of data $(\vec{Q}_{t_0}, \vec{P}_{t_0})$ to $h(t, \vec{Q}, \vec{P})$ at time t_2 in the real universe? According to the definition of $h(t, \vec{Q}, \vec{P})$, because the assumption is that this universe is deterministic, and $h(t, \vec{Q}, \vec{P})$ can surely explain the evolution to particles in the past. However, if $h(t, \vec{Q}, \vec{P})$ cannot give the prediction in t_0, then with the same input at time t_2, $h(t, \vec{Q}, \vec{P})$ cannot interpret the evolution of particles in the past, which means this universe is not deterministic because the physics mechanics explain how determinism happened does not exist.

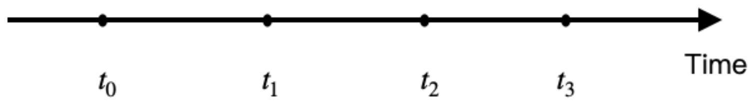

Figure 4. The Time Points of Occurrence of Events.

However, some may argue that we can never find a group of data like $(\vec{Q}_{t_0}, \vec{P}_{t_0})$, because the complicated calculation of $h(t, \vec{Q}, \vec{P})$ must lead to chaos. Hence, a very small uncertainty will cause totally different results. But this problem can be easily solve by asking if there exist a group of data $(\vec{Q}_{t_0}, \vec{P}_{t_0})$ to describe the particles in universe at t_0 or not. According to the assumption of a deterministic universe, the answer must be yes, then it can always lead to the contradiction above.

Therefore, whatever the output $h(t, \vec{Q}, \vec{P})$ gives the above experiment setup (not terminate or give the prediction). The determinism is always invalid, which means if the particles have determined position and momentum, then people can never find a mathematical interpretation to interpret how this mechanics determined the particles in the universe. Hence, it seems the interpretation Bohmian Mechanics gives us is invalid, and Copenhagen Interpretation is correct: particles do not have position or momentum until measurement occurs to them.

2.5. A Possible Valid Explanation to Preserve Determinism

There remain only one possible situation to make the determinism valid in above experiment setup in **Figure 5**, which is $h(t, \vec{Q}, \vec{P})$ does give the prediction about $g(s)$ at time t_1, but the time $h(t, \vec{Q}, \vec{P})$ give the prediction is at $t_1 - \Delta t$, where $\Delta t = \frac{\Delta x}{c}$. This statement can be proved easily.

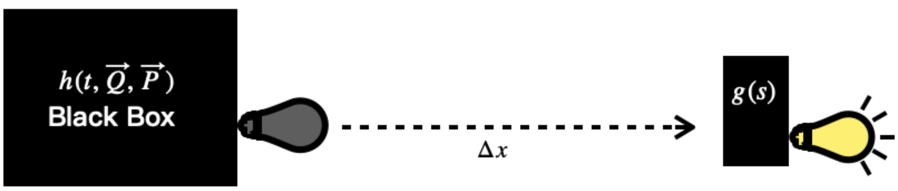

Figure 5. Physical Simulation of Halts Problem

In order to give correct prediction, the signal light on $h(t, \vec{Q}, \vec{P})$ must become same as $g(s)$. Because $g(s)$ always gives antithetical result from the detector observe the result on $h(t, \vec{Q}, \vec{P})$, if $h(t, \vec{Q}, \vec{P})$ not terminate until $t_1 - \Delta t$, then signal light on $h(t, \vec{Q}, \vec{P})$ will be on. The information needs Δt to travel to $g(s)$, at time t_1, signal light on $h(t, \vec{Q}, \vec{P})$ is on, and signal light on $g(s)$ is also on. In this case, $h(t, \vec{Q}, \vec{P})$ gives the correct prediction.

In Figure 6, it seems that $h(t, \vec{Q}, \vec{P})$ can give the prediction any time after time $t_1 - \Delta t$, which will cause contradiction. However, people can replace $g(s)$ to $G(s)$, which signal light on $G(s)$ will represent same state as light on $h(t, \vec{Q}, \vec{P})$. Then, people can find that $h(t, \vec{Q}, \vec{P})$ can give the correct prediction any time before $t_1 - \Delta t$. Hence, combine this two condition, $h(t, \vec{Q}, \vec{P})$ can give the correct prediction to certain place at time $t_1 - \Delta t$, where Δt is the time light information needs to travel to that place.

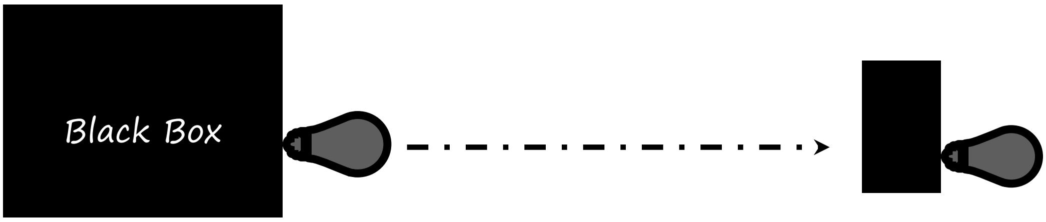

Figure 6. Physical Simulation of Halts Problem

In conclusion, this possible explanation shows that, even though there exists $h(t, \vec{Q}, \vec{P})$, like the equations Bohmian Mechanics illustrates, people have precise information about every particle, and they are able to use them to predict the future. But in this case, determinism is consistent.

3. Conclusion

In this paper, the author introduced some intuitive calculation of Heisenberg's uncertainty principle. Based on the phenomena of the uncertainty principle, the author introduced some valid interpretations of Quantum Mechanics, the Copenhagen Interpretation and the Bohmian Mechanics. However, the deterministic picture of the universe that Bohmian Mechanics reflected is really fascinating, hence the author also gives some ideas about Turing's Halting Problem. In the last part, the author combines the self-reference idea in Turing Halting Problem into the deterministic universe that Bohmian Mechanics shows. And this thought experiment shows that $h(t, \vec{Q}, \vec{P})$ cannot give the prediction of the future, but it still can give the result of what had already happened in the past, which may lead people re-think about the idea of determinism and hidden variable interpretation. According to determinism, the future is deterministic, but unpredictable even though given by the precise initial condition of particles. Furthermore, $h(t, \vec{Q}, \vec{P})$ lost its duty and cannot give the prediction of the future, our classical concept of determinism should be revised, because it seems that something determined does not mean something predictable.

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