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Two-Dimensional Ideal Fluid Dynamics and Flow Around a Circular Cylinder Based on Complex Variable Analysis

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Abstract. The paper mainly studies how to apply complex variable analysis to practical problems related to a two-dimensional ideal fluid. The author's purpose is to provide a mathematical and physical method to optimize and facilitate industries related to fluid mechanics, such as oceanography, civil engineering, the aircraft wing manufacturing field, and so on. The main mathematical methodology is complex potential, and the main physical condition is the theorem and definition of the two-dimensional ideal fluid. The result of the study is that the use of complex variable analysis is significantly effective, and the processes of the calculation are much easier for this reason. The calculation process can prove the practicability. In this paper, the author combines mathematical derivation with methods of application, and the author also combines physical definitions and physical properties applied in the paper, clearly demonstrating the method of how complex potential works in practical problems. In conclusion, complex variable analysis can be used in many different fields related to physics, and it can facilitate the development of engineering and reduce the possibility of making mistakes in calculations, making resolutions precise and lowering the cost of the processes.

Keywords: Two-Dimensional Ideal Fluid, Complex Variable, Complex Potential, Mathematical Physics, Cylinder

1. Introduction

The application of mathematics has long been a significant force, providing the foundational language and rigorous framework for the discovery of physics. Moreover, the application of sophisticated mathematical theorems and tools to physical problems always produces accurate and precise solutions, providing insights into the laws of nature. Among the infinite mathematical tools, the theory of complex variables stands out for its distinctive ability to simplify and solve problems in two-dimensional physics fields, such as fluid dynamics, heat conduction, mechanics, and electromagnetism. In potential fluid mechanics, the unparalleled methods of complex analysis provide a remarkably succinct, streamlined framework for tackling the issues of inviscid, incompressible, two-dimensional, and irrotational fluid, commonly known as ideal or potential fluid. The study of two-dimensional ideal flow not only has theoretical significance, but it also erects the foundational basis for understanding more complicated fluid behaviors in many cases. Although modern Computational Fluid Dynamics is capable of simulating turbulent and viscous flows nowadays, finding analytical solutions derived from potential fluid theory remains essential and pivotal for verifying numerical models and comprehending basic flow phenomena. This theoretical framework, therefore, finds extensive applications in solving practical engineering problems.

The practical implications of two-dimensional potential flow extend to numerous engineering disciplines. In ocean engineering, the calculations of the stability of submarine pipelines are extremely crucial. When ocean currents flow around the pipelines under the water, highly complex flow phenomena occur. Based on complex analysis and ideal fluid theory, engineers can predict the flow of fluid structures, water pressure distributions, and the potential scouring zones, and so on. Rajeev proposed that incompressible fluids in two dimensions can be applied to ocean currents and atmospheric flows, and these methods are really practical because they can simplify an extremely complex engineering problem into several understandable equations [1]. Moreover, in the aerospace field, potential flow theory of ideal fluids and complex variable analysis allows for rapid analysis of the ideal lift characteristics of airfoil sections, which remain valuable in the early stage of the design phase. Perezhogin and Academician Dymnikova found that the approximations of the equations for a viscous fluid can be practically applied in the case of asymptotic with low velocity, which is a very difficult area of fluid mechanics and plasma physics [2]. Anwar and Srigutomo analyzed that the two-dimensional ideal flow can be used to solve boundary-value problems in a set of boundary conditions [3].

The complex variable analysis effectively simplified the process of applying the potential fluid theory and ideal flow methods, and the complex functions also reduce the probability of making errors in the case of using ideal fluid theory to calculate in many significant fields. The majority of Academics and scholars research how two-dimensional Ideal fluid dynamics can be used in different scientific fields and engineering processes. However, a more focused investigation into specific classical cases, such as the flow around a circular cylinder, remains highly valuable for elucidating fundamental principles. The case of flow around a circular cylinder is a representative example of the application of ideal fluid theory, which is very helpful in understanding how the potential flow dynamics are unique and distinctive.

This paper will explore and study the application of complex analysis in two-dimensional potential fluid, following the logic and stages outlined below. Firstly, the paper will introduce the formula about complex variable analysis and some basic knowledge of complex functions (complex potential, Cauchy-Riemann Equations, Cauchy Residue Theorem, and so on), which are significant in the article. To elaborate, this paper will provide a detailed exposition of the theoretical mathematical theories that are most frequently applied, and the author will also introduce the definitions of a two-dimensional ideal fluid and some important details in the physics field that will be used in this paper (complex velocity, uniform flow, point vortex, and so on). Secondly, the author starts to research and provide how the complex variable analysis and complex function method can be applied to problems associated with fluid dynamics, and the author will emphasize and explain the overall application approach and the reason why the author uses these methods. The second general section is the most significant part of this paper, and the article will also mention the importance of these theorems in real society and in the scientific field. Thirdly, the paper will introduce and analyze the phenomenon of flow around a circular cylinder. Because this phenomenon pertains to the problem of ideal flow potential and is also a typical example, the paper will help scholars deeply understand the fluid dynamics problems based on complex variable analysis. Simultaneously, the article will illustrate some additional mathematical and physical theories (Kutta-Joukowski Theorem, Conformal Mapping, and so on). Finally, the author will provide a general conclusion about the research and problems discussed and optimize the ideas presented in the paper.

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Moreover, the author will reflect on the limitations of the application of the methods used in this article, while conducting reflections and looking for better solutions in the future. The article will also mention the importance of these theorems in real society and in the scientific field. The purpose of this work is to study how to explore and use complex analysis to tackle issues associated with practical problems in fluid physics, as well as how this mathematical method can be leveraged to a broader range of scenarios in today's entire scientific world.

2. Case study: two-dimensional potential flow around a bluff body

As for a very crucial part of physics, fluid mechanics is an absolutely indispensable field of science. Nowadays, a wide variety of two-dimensional ideal fluid theorems are extremely practical in oceanography, civil engineering, architecture design, aerospace, and so on. At the same time, numerous fields related to natural science have applied complex variable analysis to reduce some sophisticated processes during calculations. In today's society, the use of complex functions to solve ideal flow problems is profoundly convenient and widely applied in international scientific research and many engineering-associated industries. Amosova and Ozerova found that irrotational flow around a round cylinder and complex potential can be integrated and leveraged well, enabling the solution to flow-related problems to be more insightful [4]. Olver researched conformal mapping and proposed that complex analysis can transform the highly abstract concept of conformal mapping into visualizable formulas, largely simplifying the process. With the continuous development and advancement of technology and modern science in today's world, the scientific problems and challenges that individuals face have gradually become more sophisticated. Lacave discussed the application of complex analysis in the study of the properties of Jordan arcs and Jordan curves with the thin obstacle convergence problems, which are specific examples of two-dimensional ideal fluid, and he proposed that these abstract and sophisticated problems can be simplified using complex variable analysis [5,6]. As the manufacturing of equipment and devices becomes more and more common, utilization rates have grown rapidly in engineering, oceanography, computer technology, and certain aerospace fields. Adopting faster, simpler, and more efficient methods to solve complicated and intricate scientific problems within is a crucial step. In conclusion, the flow motion of two-dimensional ideal fluids and the application of complex variable analysis play a significant role in engineering or science-related research projects in today's studies, and the combination of these two has created an extraordinary high value in a multitude of domains. Viewed separately, the author considers that the complex variable analysis and the ideal fluid theorem also have great value and reflect the integration of mathematics and physics across many sectors. Utilizing mathematical theorems that were previously used in theory in physics plays an extremely important role in facilitating the development and progress of the whole scientific community. To summarize the basic information based on the case that this paper studies, the application of this research has long been recognized and gained acceptance in society. The author regards this science as an exploration with practical significance and value, whether in very fundamental fluid mechanics problems or extremely difficult engineering challenges in the entire world; these fields make pivotal contributions to society, thereby reflecting the social significance brought by this study. The analysis of this specific case will serve to demonstrate the core methodologies of complex analysis in fluid dynamics and will provide the foundation for understanding more complex geometries through techniques like conformal mapping.

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3. Analysis of the problem

3.1. Problem analysis and basic theoretical concepts

3.1.1. Complex functions

A complex function is a function whose domain is the field of complex numbers, which maps a complex number to another complex function. The domain and range of a complex function are both complex number sets.

A complex function can be expressed by:

$$f(z) = w, z = x + iy \tag{1}$$

Z is a complex number, and $z = x + iy$ is a basic expression of a complex number. x is the Real Part of the complex number, and y is the Imaginary part of the complex number.

The expression of w: $w = u\,(x, y) + iv\,(x, y)$, $u\,(x, y)\; and\; v\,(x, y)$ are real functions, representing the real part and the imaginary part of the complex function, respectively. A complex function can be either a single-value function or a multiple-value function. If each complex number z is corresponded to exactly one complex number w, this complex function can be regarded as a single-value function; if each complex number z is mapped to many different complex numbers w, this function can be qualified as a multiple-value function.

3.1.2. Differentiability and the cauchy-riemann equations

The necessary and sufficient condition for the function $f\,(z) = u\,(x, y) + iv(x, y)$ to be differentiable at the point (x, y) is:

$u\,(x, y)$ and $v\,(x, y)$ can be differentiable at the point (x, y) .

$u\,(x, y)$ and $v\,(x, y)$ have to satisfy the Cauchy-Riemann Conditions at the point (x, y) .

The Cauchy-Riemann Conditions are:

$$\frac{du}{dx} = \frac{dv}{dy} \text{ and } \frac{du}{dy} = -\frac{dv}{dx} \tag{2}$$

These conditions serve a vital function in subsequent sections of this paper, because they can perfectly align with the physics theorems about a two-dimensional ideal fluid that the author discussed later.

3.1.3. Two-dimensional ideal flow: assumptions and simplifications

Ideal fluid is a theoretical fluid model in Fluid Mechanics. It has four traits: The model studied in this paper further assumes the flow is two-dimensional and steady.

Inviscid: it means that there is no internal friction and there is no viscosity.

Incompressible: it means that the density of the fluid is constant and the fluid cannot be compressed.

Irrotational: It means that fluid particles do not rotate around their center of mass, and it also means that the vorticity is equal to zero.

Because there is no internal friction in the ideal fluid, there is no heat conduction at the same time.

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There are two characteristics of Two-Dimensional fluids:

Two-Dimensional Flow: It means that the motion of the fluid occurs in the x-y plane, and the velocity field is described in a certain x-y plane.

Two-Dimensional Steady Flow: It means that the velocity field of the fluid does not change with time.

The definitions of steady flow and two-dimensional flow are distinct concepts that, when combined, define a two-dimensional steady flow.

The usual way of describing a fluid flow is to describe it by means of the flow velocity defined at any certain point $p = (x, y, z)$ at any time in terms of t. The author believes that the world is three-dimensional in real life, and people should first comprehend the definition from a three-dimensional perspective.

$$u = u(p,t) = [u(p,t), v(p,t), w(p,t)] \tag{3}$$

Kundu and Cohen analyzed that (x, y, z) are three velocity components in the three Cartesian directions can be indicated by $(u,\ v,\ w)$, in a three-dimensional system [7].

According to this information and definitions:

Because of the definition of the two-dimensional flow, the motion of the fluid must occur in the x-y plane. This means that the velocity field analysis along the z-axis does not exist. In terms of mathematical language, this means that the velocity component w that represents the velocity in the z-axis direction does not exist, $w\ (\ p,\ t\ ) = 0$.

The two-dimensional flow can be regarded as this term:

$$u = u(p,t) = [u(p,t), v(p,t), 0], \text{where } p = (x, y) \tag{4}$$

Because of the definition of steady flow, the velocity field of the fluid does not change with time. In terms of the mathematical language, if there is a function, its x-axis is time t and its y-axis is the flow velocity, the graph of its function will appear as a straight line parallel to the x-axis. This means that the slope of this function's graph remains zero.

According to this information, the author thinks that the definition can be represented in the following terms:

$$\tfrac{du}{dt} = 0,\ \text{where } u = u(p) = [u(p), v(p), w(p)] \tag{5}$$

After these steps, they can be put together and become the mathematical expression of the definition of Two-Dimensional Ideal Fluid:

$$u = [u\,(p),\ v\,(p),\ 0\,],\ \textit{where the point } p = (x, y) \tag{6}$$

3.2. The need for a complementary approach: limitations of traditional methods

3.2.1. The established value of traditional methods

Numerical, theoretical, and experimental methods are indispensable pillars of fluid mechanical research. However, when applied in isolation to the problem of two-dimensional potential flow, each faces significant challenges.

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Numerical computation methods can be used to obtain specific numbers and detailed values for sophisticated problems that people can't solve by analyzing them. It transforms abstract problems into specific numerical problems, promoting estimation and processes in the engineering field at relatively low costs.

Theoretical analysis methods allow people to clearly identify the variation relationship between individual physical quantities and different flow parameters, and researchers can use this characteristic to draw many valuable functional figures that can be used in many fields of engineering.

Experimental methods are significantly intuitive. They can be used to facilitate the resolution directly, and scientists can also have opportunities to discover new principles and phenomena. In the fields of science, the experimental approaches have long been regarded as the most credible and compelling methods.

Wang analyzed that the scientific community recognizes these three methods and has been using them for decades, and he also found that the majority of crucial concepts and fundamental principles are derived from experimental research [8].

3.2.2. The established value of traditional methods

Although these three methods all have their special advantages, the author believes that their inherent limitation and shortcomings are obvious.

Numerical computation methods are only computation itself in some cases. When addressing very complicated problems without a well-established mathematical model, numerical computation approaches are not able to tackle these issues. Meanwhile, numerical computation methods are susceptible to errors. If there are constraints on computation resources and limitations on computation accuracy, it becomes difficult to solve all relevant problems.

Although theoretical analysis approaches are applicable, the difficulties of mathematical computations are immense. Pure theoretical analysis often requires solving Laplace's equation subject to complex boundary conditions, which can be mathematically formidable and yield closed-form solutions only for simple geometries.

While the experimental methods are widely recognized as the most applicable methods in scientific research, the costs of human resources, financial resources, and material resources for different depths of experiments are extremely high.

4. Suggestions

The limitations outlined in Section 3.2 highlight the need for a powerful analytical framework that can provide exact solutions for a class of flow problems, thereby complementing traditional approaches. The theory of complex variables provides this framework. To fix the problem mentioned in 3.2.2, the author proposes combining complex variable analysis with two-dimensional ideal fluid mechanics as an effective and reasonable solution to these shortcomings and limitations. To be specific, many of the constraints of differentiability in complex variable functions are congruent with two-dimensional ideal fluid mechanics. This section is the main part of the paper, and this part reflects the significance and advantages of the research associated with how to use the complex variable analysis method as a distinctive tool to solve fluid mechanics problems. The critical step in applying complex variable analysis to ideal fluid mechanics is to give the physics meaning to the complex function.

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4.1. The complex potential: linking mathematics and physics

Firstly, the basic form of a complex function is:

$$z = x + iy \tag{7}$$

The analytic function of z is：

$$f(z) = u(x, y) + iv(x, y) \tag{8}$$

For two-dimensional potential flow, this analytic function takes on profound physical significance. It is termed the complex potential, where in the real part $u(x, y)$ is identified as the velocity potential ϕ, and the imaginary part $v(x, y)$ is the stream function ψ.

The next part is associated with Methods of Mathematical Physics. Complex functions can describe the motion of an ideal fluid as the basic term of the complex function. In the complex function, the real part of it is the velocity potential function, and the imaginary part of it is the stream function. Hildebrand found that using the analytic function of a complex number z corresponds to the flow velocity. V_x and V_y is reasonable [9]. The key part is how to apply the correspondence between the real part, the imaginary part, the velocity potential function, and the stream function, and use their relationship to show V_x and V_y.

4.2. Application example: uniform flow and the complex potential

4.2.1. Basic application and expression of methodologies in mathematical physics

In a two-dimensional ideal fluid, complex functions can describe the motion of an ideal fluid (inviscid, irrotational, incompressible fluid). The author believes that only combining theoretical mathematics and physics can make individuals understand and solve practical problems intuitively and directly. Therefore, the author intends to illustrate the advantages of applying complex variable analysis to two-dimensional ideal fluid mechanics through very specific examples, and this example is one of the most paradigmatic examples in fluid mechanics: Uniform Flow Past a Circular Cylinder.

Firstly, based on the complex function mentioned in 3.1.1 and the definition of two-dimensional ideal fluids in 3.1.2, the author considers establishing a connection between these two concepts before describing the specific problem. In ideal fluid mechanics, the complex potential function $W(z) = \phi + i\psi$, and what ϕ and ψ represent respectively, have been mentioned above in 4.1.

Why is the basic form of a complex function extremely effective? There are several reasons:

1. Two-dimensionality: The complex point $z = x + iy$ on the complex plane can perfectly correspond to a point (x, y), which is a useful condition to solve the practical problem.

2. Two traits of ideal fluid, irrotationality and incompressibility, can totally correspond to the Cauchy-Riemann equation in theoretical mathematics, which allows people to define a vital composite function: complex potential.

To be specific, the author considers a two-dimensional motion, and the velocity field is

$$V = (u(x, y),\ v(x, y)) \tag{9}$$

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The physical meaning of incompressibility is that the density of the fluid is constant and the fluid cannot be compressed, and the mathematical meaning of it is that the divergence of the velocity remains zero. $\nabla V = 0$ can be represented as $\frac{du}{dx} + \frac{dv}{dy} = 0$ in a two-dimensional Cartesian coordinate system. The physical meaning of irrotationality is that fluid particles do not rotate around their center of mass, and it also means that the vorticity is equal to zero. Mathematically speaking, it means that the curl of the velocity remains zero. In addition, in a two-dimensional case, the curl has only one component of z, therefore, the condition $\nabla V = 0$ can be represented by $\frac{du}{dx} - \frac{dv}{dy} = 0$. King, Billingham, and Otto researched the combination of these physical and mathematical meanings of ideal fluid, which is very effective and can be used easily for simple flow [10].

4.2.2. Citation of potential function and stream function

In vector calculus, any vector field for which the curl is zero must be the gradient of a scalar function. This is really complex if it can only be understood by words, so in mathematical explanation:

If $\nabla V = 0$, the function $V = \nabla \phi$ must exist.

Also, in the two-dimensional case, it has:

$$u = \frac{d\phi}{dx} \text{ and } v = \frac{d\phi}{dy} \tag{10}$$

The function $\phi(x, y)$ is the velocity potential function.

Similarly, any vector field for which the divergence is zero in vector calculus can be expressed as the curl of another vector field. In the two-dimensional case, it means that:

If there is a function $\psi(x, y)$, there are two conditions that can be satisfied

$$u = \frac{d\psi}{dy} \text{ and } v = -\frac{d\psi}{dx} \tag{11}$$

The function $\psi(x, y)$ is called the stream function. Its physical meaning is extremely significant because the contour of the stream function exactly streamlines when individuals solve practical problems. Moreover, the most fundamental mathematical expression of the defining equation of a streamline is:

$$d\psi = \frac{d\psi}{dx} *dx + \frac{d\psi}{dy} *dy = -vdx + udy = 0 \tag{12}$$

Horace proposed that this calculation is based on the same velocity field $V = (u, v)$, and because of their relationship, many sophisticated equations like the equation of a streamline can be derived [11].

In conclusion, there are four different equations from the traits of irrotationality and incompressibility, as shown in Table 1:

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Table 1: Four Different Equations

From irrotationality	From incompressibility
$u = \dfrac{d\phi}{dx}$	$u = \dfrac{d\psi}{dy}$
$v = \dfrac{d\phi}{dy}$	$v = -\dfrac{d\psi}{dx}$

Let:

$$u = \frac{d\phi}{dx} = \frac{d\psi}{dy} \text{ and } v = \frac{d\phi}{dy} = -\frac{d\psi}{dx} \tag{13}$$

This is the famous Cauchy-Riemann equation in complex function theory.

4.3. Example analysis: flow around a circular cylinder

4.3.1. Basic flow elements

In complex variable analysis, there is a critical application: conformal mapping. Through conformal mapping, an enormous amount of complicated shapes and objects can be mapped to elementary shapes, thereby solving the related problems effectively. Polya and Latta proposed that using the Schwarz-Christoffel mapping can map the flow around the cylinder problem to the upper half plane, so that the difficult computation can be simplified [12].

Before solving the problem about the flow around a cylinder, the author believes that people need to understand that the superposition of the following basic flow elements can form any complicated irrotational flow:

1. Uniform Flow: The complex potential of the uniform flow with velocity V and with an angle α with the positive real axis x can be defined as $w(z) = Ve^{-i\alpha}z$. When the direction of the motion of the flow is horizontally to the right, it means that the angle α is equal to 0, the complex potential can be simplified as $w(z) = Vz$. Corresponding to the velocity potential function ϕ and the stream function ψ , the author thinks:

$$\phi = Vx \text{ and } \psi = Vy \tag{14}$$

2. Source and Sink of Complicated Irrotational Flow: The distinction between a point source and a point sink is the value of the strength Q at the origin of coordinates. The complex potential of a point source that $Q > 0$ or a point sink that $Q < 0$ can be defined as: $w(z) = \frac{Q}{2\pi} lnz$. The streamlines of the flow are a family of rays that radiate outward from the origin or converge toward the origin. For the point source, they radiate outward; for the point sink, they radiate inward.

3. Dipole: The complex potential of a dipole located at the origin with a strength μ and a direction angle - α is: $w(z) = \frac{\mu e^{i\alpha}}{2\pi z}$. The direction of the angle - α points from the point sink to the point source. The dipole refers to a limiting flow model that forms when a point sink of equal

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strength and a point source approach infinitely close to each other in a certain space. The strength and the direction of the angle are definite and clear, and the dipole is a core element that solves the problem of flow around the cylinder.

4. Point Vortex: The complex potential of a point vortex located at the origin with a circulation Γ is: $w(z) = \frac{i\Gamma}{2\pi} lnz.$ The streamlines of the flow are concentric, centered at the origin; the sign $(+or-)$ of the circulation Γ determines the direction of the ideal fluid that rotates around the origin.

4.3.2. The specific solution of the problem

The author constructs a fluid that flows with the velocity V and flows along the x-axis direction. The fluid now passes an infinitely long cylinder with radius a. The motion of the flow can be regarded as the sum of two simple motions of fluid:

1. The uniform flow that flows along the positive x-axis direction: $w_1(z) = Vz$.

2. The dipole that is along the negative x-axis direction at the origin: $w_2(z) = \frac{\mu}{2\pi z}$ (because α is 0), in this case, the value of μ is to be determined.

To combine them, the total complex potential can be defined as:

$$w(z) = w_1(z) + w_2(z) = Vz + \frac{\mu}{2\pi z} \tag{15}$$

After knowing the total complex potential, the next step is to determine the strength μ of the dipole and to verify the boundary of the cylinder. The boundary of the cylinder is a streamline. Because the fluid cannot pass through the cylinder in the real case, the surface of the cylinder itself is a streamline. $\psi = 0$ can be used as the boundary condition, which means that $\psi = \mathrm{Im}[\mathrm{w}(\mathrm{z})] = 0$ exists on $|z| = a$, and it can be used to determine the strength μ .

The exponential form of the complex number is $z = re^{i\theta}$, it can be used in the complex potential:

$$w(z) = Vz + \frac{\mu}{2\pi z} = Vre^{i\theta} + \frac{\mu}{2\pi re^{i\theta}} = Vre^{i\theta} + \frac{\mu}{2\pi r}e^{-i\theta}$$

$$= \left(Vr + \frac{\mu}{2\pi r}\right)cos\theta + i\left(Vr - \frac{\mu}{2\pi r}\right)sin\theta \tag{16}$$

Therefore, the stream function is the imaginary part:

$$\psi(\mathrm{r},\ \theta) = \left(Vr - \frac{\mu}{2\pi r}\right)sin\theta \tag{17}$$

In this case, the radius of the cylinder is a , it means that $\mathrm{r} = a$, and $\psi = \mathrm{Im}[\mathrm{w}(\mathrm{z})] = 0$.

So: $\left(Va - \frac{\mu}{2\pi a}\right)sin\theta = 0$ exists for all value of θ . It means that $Va - \frac{\mu}{2\pi a} = 0$, And $Va = \frac{\mu}{2\pi a}$, $\mu = 2\pi Va^2$.

Now, the author believes the strength μ is clear and can be used in the total complex potential: $w(z) = Vz + \frac{\mu}{2\pi z} = Vz + \frac{2\pi Va^2}{2\pi z} = Vz + \frac{Va^2}{z} = V\left(z + \frac{a^2}{z}\right)$, and the stream function can be represented by: $\psi(\mathrm{r},\ \theta) = \left(Vr - \frac{2\pi Va^2}{2\pi r}\right)sin\ \theta = \left(Vr - \frac{Va^2}{r}\right)sin\theta = V\left(r - \frac{a^2}{r}\right)sin\theta$

To verify whether the processes are reasonable, the author conducted an analysis of fluidity. There are many stagnation points when the fluid flows around a cylinder. The stagnation point is the

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point at which the velocity remains zero. Firstly, the complex velocity is:
$\frac{dw}{dz} = V - a^2z^{-2}V = V\left(1 - \frac{a^2}{z^2}\right)$, and the complex velocity is zero: $V\left(1 - \frac{a^2}{z^2}\right) = 0$.

It is clear that $z^2 = a^2$, so that $z = \pm a$. Using the complex variable analysis is much easier than solving the function that $u = 0 \, and \, v = 0$. Because $z = \pm a$, the stagnation points are located on the front and back edges of the cylinder.

For the surface of the cylinder $z = ae^{i\theta}$ and the total complex potential is $w\left(z\right) = V\left(z + \frac{a^2}{z}\right)$. The complex potential is $\frac{dw}{dz} = V - Va^2z^{-2} = V\left(1 - \frac{a^2}{z^2}\right)$, so that:

$$\frac{dw}{dz} = V\left(1 - \frac{a^2}{\left(ae^{i\theta}\right)^2}\right) = V\left(1 - \frac{1}{e^{i2\theta}}\right) = V\left(1 - e^{-i2\theta}\right) \tag{18}$$

Because most discussions of this problem focus on the magnitude of the complex velocity, the author considers that people should find out the absolute value of the complex velocity to exclude its direction, and the direction is never a key point in this case.

$$\because e^{-i\beta} = cos\beta - isin\beta, \text{ let } \beta = 2\theta,$$

$$\therefore e^{-i2\theta} = \cos\left(2\theta\right) - i\sin\left(2\theta\right)$$

$$\frac{dw}{dz} = V\left(1 - \cos\left(2\theta\right) + i\sin\left(2\theta\right)\right) \tag{19}$$

The real part is: $V\left(1 - \cos\left(2\theta\right)\right)$; the imaginary part is: $V\left(\sin\left(2\theta\right)\right)$

$$\left|\frac{dw}{dz}\right| = \sqrt{\left[V\left(1 - \cos\left(2\theta\right)\right)\right]^2 + \left[V\left(\sin\left(2\theta\right)\right)\right]^2}$$

$$= V\sqrt{\left[\left(1 - \cos\left(2\theta\right)\right)\right]^2 + \left[\left(\sin\left(2\theta\right)\right)\right]^2}$$

$$= V\sqrt{4sin^2\left(\theta\right)} = 2V\left|\sin(\theta)\right| \tag{20}$$

Because the stagnation points are located on the front and back edges of the cylinder, it means that the velocity of the flow is the highest. The range is from $\theta = \frac{\pi}{2} \;\; to \, \theta = \frac{3\pi}{2}$, $\left|\sin(\theta)\right| = 1$ always exists, so that $\left|\frac{dw}{dz}\right| = 2V$. Overall, this phenomenon conforms to Bernoulli's principle: when the velocity increases, the pressure will decrease.

If the cylinder rotates when the flow passes the cylinder at the same time, the circulation Γ will exist, and it means that the flow will no longer be symmetric. The problem will be extremely difficult in this situation because it is special in the flow around the cylinder. But the complex potential can solve this problem effectively: it is only necessary to superimpose the complex

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potential of a point vortex located at the origin onto the complex potential of the flow without circulation Γ : $w(z) = V\left(z + \frac{a^2}{z}\right) - \frac{i\Gamma}{2\pi} lnz$, the function of i is rotation.

After analyzing the specific problems, a principle can be applied to this problem- the Kutta-Joukowski Theorem. This principle is not only associated with the fluidity of the flow, but also related to the lift force L . Munson, Okiishi, Huebsch, and Rothmayer proposed that the Kutta-Joukowski Theorem is a specific application of the complex potential in differential analysis of fluid flow in many difficult cases [13].

Because the circulation Γ exists, the fluid flow is not symmetric. Therefore, stagnation points can move from $z = \pm a$ instead of staying in a point, and it is possible that the stagnation points can merge together. Kutta-Joukowski Theorem is a critical and basic theorem in the fields of fluid mechanics: the lift force that acts on a certain cylinder per unit length is $L = \rho V\Gamma$, ρ is the density of the fluid, V is the fluid velocity, and Γ is the circulation of the flow. When the velocity V rotates by 90 degrees against the direction of the circulation Γ , the resulting direction is the direction of the lift force L . The relationship between the velocity, circulation, and the density can be applied to solve practical problems in scientific research effectively.

5. Conclusion

The paper mentions three different reasonable methods to solve the problems associated with the two-dimensional ideal fluid: numerical, theoretical, and experimental methods. Even though these three methods have many unique advantages, their limitations cannot be ignored. In order to solve the problem with the minimum level of limitation, the author suggests that individuals can combine the complex variable analysis with the ideal fluid problems. The paper mentions the basic roles, definitions, and formula at first, and then showcases the effectiveness and the necessary functions of the complex potential with details of calculations. From a microscopic perspective, the method is the application of complex variable analysis to fluid motion; from a macroscopic perspective, it represents the perfect combination of mathematical and physics methods. The purpose of the mathematical methods used in this paper is to show the immense power of complex variable analysis in the field of fluid mechanics and to provide recommendations for the application of complex potential in relevant industries in today’s society.

However, the complex variable analysis has its own limitations. The complex potential can be completely constructed based on an ideal fluid, which means that inviscid flow is an extremely vital condition. Therefore, the complex potential method actually ignores that the fluids in the real world all have the characteristics: viscosity, compressibility, and fluidity. Ideal fluids do not exist in the real world. In conclusion, the application of complex analysis and theorems of two-dimensional ideal fluid can only be used to make a precise estimation, but it cannot truly solve problems directly. Moreover, this paper only analyzes one example: flow around the cylinder, and it does not fully present or search for the specific data or images from an actual experiment, so the paper itself also has certain limitations. Of course, with the continuous development of science and technology, the popularity of theoretical mathematical methods will be enhanced in the future, and strategies of using complex analysis will be optimized.

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