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Formal topological description of three-dimensional topological entities based on k-dimensional pseudo-manifold

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Abstract. This paper clarifies the necessity of studying three-dimensional spatial data models, analyzes the research progress and existing problems of topological spatial relationship description methods, and proposes a complete and formal description framework of three-dimensional topological spatial relations based on point set topological theory and dimension expansion method. On this basis, the topological spatial relations existing in three-dimensional spatial objects are classified, and five fundamental topological spatial relations are defined. The mutual exclusivity and completeness of the minimal set of 3D topological space relations are proved. Based on point set topology and k-dimensional pseudo-manifold, this paper proposes the formal description of three-dimensional topological properties and spatial relations between spatial entities. The simplicial homology is applied to manifolds, the topological properties of three-dimensional space entities are revealed, and the formal description method of three-dimensional space entities is given through the geometric structural relationship between simplex and simplicial k-complex.

Keywords: composite topology, point set topology, 3D space entity, topological properties, formal description.

1. Introduction

The formal description method of spatial entities and their topological properties directly affects the integrity and effectiveness of spatial data organization, spatial query, spatial reasoning and spatial analysis. In the research process of two-dimensional spatial data model, many scientists have discussed the definition of two-dimensional space and spatial entities from different perspectives, such as spatial data organization, expression of spatial relations between objects, and data integrity constraints. Among them, the representative system is the topological unit structure theory proposed by Corbett and Whito [1], that is, the topological theory and graph theory on two-dimensional Euclidean space are used to explain and describe two-dimensional spatial entities and the spatial relations between entities. In this method, the real world is abstractly expressed as a 2D Planar Graphs G - (v, R). Any spatial entity in the two-dimensional Euclidean space can be regarded as a subgraph G 'of the graph G, which corresponds to a graph G' in the two-dimensional topological space. According to its spatial dimension, Mou Chun's complex spatial entity can be divided into point entity (Oomplex), line entity (LineEntity Lomplex) and surface entity (Surface Entity). The geometric structure of any two-dimensional plan can be described by two-dimensional simple complex [2]. Compared with the two-dimensional situation, is the semantic concept and formal description of three-dimensional spatial

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entities much more complex? On the one hand, because the spatial objects in the three-dimensional real world are diverse in form and characteristics, and there are extremely complex spatial relationships between them, it is difficult to reasonably classify these spatial objects and give a complete semantic concept; On the other hand, because the complex and diverse spatial objects in the real world involve a large amount of three-dimensional spatial information, it is difficult to effectively express and store these information in a digital form, and display them on a two-dimensional computer screen. It can be seen that the semantic concept and formal description of 3D spatial entities are difficult problems in the field of spatial information and many related disciplines.

Corbett, Simon Pigot and others expanded the two-dimensional topological unit theory, discussed the definition and description of three-dimensional spatial entities and their topological properties, and proposed the concept of three-dimensional topological space unit theory [3]. They define a space as a manifold (3-manifold 1d) space, and each spatial entity corresponds to one or more connected or separated k-manifolds. Among them, they introduce the topological properties of manifolds into spatial entities as an expression tool of topological properties such as connectivity direction and identity of spatial entities [4]. These methods are more innovative than the two-dimensional topological unit theory both in theory and in implementation, but they have the following problems: first, there is a lack of strict definition of the semantic concept of spatial entities, and there is a lack of strict mathematical description of concepts such as space, the dimension of spatial entities, and the division of spatial entity types; Secondly, there is a lack of strict derivation, proof and expression mechanism for topological properties of spatial entities, and a lack of in-depth discussion on the corresponding relationship between spatial entities and manifolds, simplexes and simple complexes; Furthermore, the formal description method of spatial entities and their geometric structures is not given, which brings a series of difficulties to the expression of spatial relations and the design of data structures; Finally, there is a lack of relevant experimental guidance and geometric implementation of spatial data model. It can be seen that there are still a lot of problems to be further discussed and solved in the research of formal description of three-dimensional space entities. In this paper, based on the combination topology and point set topology theory, a new definition of the semantic concept of three-dimensional spatial entities based on manifold topology is proposed. On this basis, the topological properties and formal description methods of three-dimensional spatial entities are derived, which lays the foundation for establishing a three-dimensional topological data model that can effectively express spatial entities and their topological spatial relationships.

2. The theoretical basis of point set topology

The so-called metric space refers to the set R with any element (point) and metric introduced into the abstract set [5]. For any two points of the set x, y determines the distance p (x, y) between them and satisfies the following axiom of the metric space:

(1) $p(x,y) > 0$, When $x \neq y$; $p(x,x) = 0$

(2) $p(x,y) = p(y,x)$ (Axiom of symmetry)

(3) $p(x,y) + p(y,z) \geq p(x,z)$ (Triangle inequality) The set R forms a metric space.

The so-called topological space is the set X of elements (points) that meet the following conditions. For each element (point) x of R, a non empty group with a subset of X as a member is selected. This subset is called a neighborhood of x, and meets the following topological space axioms:

(1) X is in every field of its own;

(2) The intersection of any two neighborhood of x is a neighborhood of x;

(3) If N is the neighborhood of x and U is the subset of X containing N, then U is the neighborhood of x;

(4) If N is the neighborhood of x, And if N° represents a set $\{z \in N \mid N$ is the neighborhood of z, then N° is the neighborhood of x, and the set N° is called the interior of N. The adjacent set theory makes the adjacent metric concept generalized. A topology about R obtained from a certain degree of R, d, is called the metric topology defined by d [6]. It can be seen that every metric space is also a topological space, but the opposite is inaccurate. That is, there is such a topological space, which

cannot be made into a metric space The remainder is a set $\{x \mid x \in X \text{ and } x \notin N\}$, expressed as $X \mid N$. Point X is called the boundary point of set N. If it is neither the interior point of set N nor the interior point of its coset $X \mid N$., the set of all boundary points is called the boundary of set N, which is recorded as LN [7].

Let X and Y be topological spaces, and the mapping f: $X \rightarrow Y$ be continuous. If for every point x of X and any neighborhood N of f (x) in Y, the set f-1 (N) is the neighborhood of x in X, then the mapping f: $X \rightarrow Y$ is called a homeomorphism. If the mapping is a one-to-one continuous surjection and has a continuous inverse mapping, then X is said to be homeomorphic to Y, or X is topologically equivalent to Y. Under homeomorphism, the properties of topological space can be maintained, that is, when a property is possessed by a topological space, it is possessed by every homeomorphic space. This property is called topological invariant, and topological space relationship refers to topological invariant under topological transformation.

3. Formal definition of 3D spatial solids based on k-dimensional pseudomanifold

3.1. Topological properties of 3D spatial entities

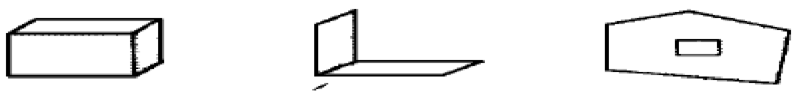
It can be seen that every metric space is also a topological space, but the opposite formulation is inaccurate. That is, there is such a topological space, which cannot make it a metric space. Therefore, Studying 3D spatial data model can define 3D spatial entities and topological spatial relationships between entities in 3D topological space [8]. Generally speaking, spatial entities have two types of attributes: geometric attributes and non geometric attributes. Their non geometric attributes can be described by relation tables; While geometric attributes can be described by absolute position information such as spatial coordinates, or by relative position information such as topological relationship, distance, direction, etc. Spatial entities can be divided into real entities according to their attributes. The entity type is a set of similar entities with the same characteristics and properties. The set of identical entities constitutes the entity set. According to the theory of composite topology, spatial entities can be divided according to their spatial dimensions. According to the definition of Ulysson dimension, spatial entities in three-dimensional topological space are composed of frequent figures whose dimensions are less than or equal to. According to their different spatial dimensions, three-dimensional spatial entities can be divided into phosphorus solids, that is, point shaped entities, linear entities, area shaped entities and body shaped entities. Among them, point shaped entities are zero dimensional graphics, linear entities are one-dimensional graphics, area shaped entities are two-dimensional graphics, and body shaped entities are three-dimensional graphics.

According to the definition of manifold and its topological properties (such as local compactness, road connectivity, directionality, etc.) [9], considering the realizability of the real world, it corresponds to a compact, connected n-dimensional manifold or a compact, connected $(n+1)$ - dimensional manifold with one or more n-dimensional manifold boundaries $(0 \leq n \leq 2)$ [10]. From this, we can define the following four kinds of spatial solids: First, body Entity: A body entity corresponds to a orientable 3-dimensional pseudo manifold. It includes two cases: one is a directed 3-dimensional manifold with closed and connected 2-dimensional manifold boundary, that is, a space body enclosed by several connected closed surfaces, as shown in Fig. 1 (a) and (b); The other is a directed 3-dimensional manifold with n $(n \geq 2)$ unconnected closed, connected and directed 2-dimensional manifold boundaries, that is, a body with internal cavities, as shown in Figure 1 (c).

(a) (b) (c)

Figure 1. Solid body [10].

Second, surface Entity: A surface entity corresponds to a orientable 2-D pseudo manifold. It includes three cases: one is a closed, connected and directed 2-dimensional manifold, which forms the closed surface of a volume, as shown in Figure 2 (a); The second is a connected and directed 2-dimensional manifold with a 1-dimensional manifold boundary, which forms part of the surface of a volume or the planar entity itself, as shown in Figure 2 (b); The third is connected and directed 2-D manifolds with multiple disconnected 1-D manifold boundaries, that is, planar entities with internal cavities, as shown in Figure 2 (c).



(a) (b) (c)

Figure 2. Faceted entity [11].

Third, line Entity: The line entity corresponds to a orientable 1-dimensional pseudo manifold, which includes two cases: one is a connected and directed 1-dimensional manifold, which forms a closed ring, as shown in Figure 3 (a); The second is a directed 1-dimensional manifold with several 0-dimensional manifold boundaries, which forms one or more connected and directed arcs, as shown in Figure 3 (b).

Fourth, pointEntity: A point is a 0-dimensional pseudo manifold, which corresponds to a spatial point with spatial location but no spatial expansion [12]. Its location information is composed of spatial coordinates X, Y, and Z. The definition of three-dimensional spatial entities based on k-dimensional pseudo manifold can achieve the following purposes: First, it highly summarizes the common characteristics of spatial objects in the three-dimensional real world, which makes the definition of three-dimensional spatial entities unified in semantic concepts, and avoids the defects such as incompleteness and imperfection caused by the simple listing method. Moreover, it is more concise and clear in expression; Secondly, it is helpful to introduce homology theory into manifolds [13]. Through the topological properties of pseudo manifolds, the topological properties of three-dimensional spatial entities can be deduced, which provides a basis for the integrity or topological consistency test of the system; Furthermore, through the structural relationship between manifold and pseudo manifold, simplex and simple complex, the geometric structure relationship of 3D spatial entities can be revealed, which is conducive to the formal description of 3D spatial entities and lays the foundation for the construction of 3D topological spatial data structure and data model.

(a) (b)

Figure 3. Linear entity [11].

The three-dimensional space entity corresponds to the orientable k-dimensional pseudo manifold, which is an important research object in topology and has many important topological properties. Using these properties of the pseudo manifold, the topological properties of the three-dimensional space entity can be deduced [14], which can be used as a systematic topological consistency checking tool and the cornerstone of designing spatial data structure.

Table 1. Topological Properties of 3D Spatial Entities.

Nature	characteristic
Simplicity	Finite number of simplexes
Directionality	An oriented simplex
Connectivity	Vertex set

In terms of simplicity, suppose that a topological space is homeomorphic to a group of finite simplexes that are well pieced together in a Euclidean space, that is, if two simplexes intersect, their common part must be a common plane, then the topological space is called simplyable. A simple partition of topological space X is composed of a simple complex Cn and a homeomorphism h: $|Cn| \rightarrow X$, which is denoted as $X = (Cn, h)$. Therefore, the simple partition of a space is not unique, it depends on the choice of simple complex Cn and the choice of partition homeomorphism h. A space that can be simply partitioned must be a metrizable compact space connected by local roads. According to the definition of three-dimensional space entity, any space entity corresponds to a orientable k-dimensional pseudo manifold $(0 \leq k \leq 3)$[15]. Because k-dimensional pseudo manifold is a simple complex with good simplex structure, it can be simply partitioned. Therefore, the three-dimensional space entity defined in this paper can be simply divided.

As for directionality, simplex has directivity. A directed simplex is designated as a directed simplex, which is recorded as $Sn = < V0i, V1i, \ldots, Vni >$, and a directed simplex that is opposite to it is recorded as - Snn simple complex Cn, which is the union of a group of finite multiple simplexes in Euclidean space Rn. If each simplex in Cn can be uniformly oriented, that is, each $(n-1)$ - dimensional simplex is induced to the opposite order by two associated n-simplexes, The simple complex is said to be orientable. It can be seen from the definition of three-dimensional space entity that the space entity corresponds to the orientable k-dimensional pseudo manifold $(0 \leq k \leq 3)$. Therefore, the three-dimensional space entity is also orientable, and all simplexes obtained by simple subdivision can be uniformly oriented. It can be seen from the concept of correlation coefficient that the sum of correlation coefficients of all simplexes contained in any 3D space entity must be 0 [16]. This is an important topological property, which can be used as a very suitable topological consistency checking tool in the process of system design and update.

In terms of connectivity, for n-simple complex Cn, let DCn represent its vertex set. a. $B \in DCn$, if $a = b$, or $a \neq b$, there are $a1, a2, \ldots, ak \in DCn$, so that (a, a1), (a1, a2),..., (ak, b) are 1-simplex of Cn, then a and b are said to be connectable, and a simple complex that any two vertices can be connected is called connected. According to the simple homology theory of topology, the homology group of a simple complex can characterize the number of connected branches of its associated polyhedron, that is, the homology group can be used to judge whether a simple complex has connectivity. Because the three-dimensional space entity is a k-simple complex $(0 \leq k \leq 3)$ with good simplex structure, it can also use its homology group as a tool to test and judge the connectivity of three-dimensional space entities. The three-dimensional space entity corresponds to a orientable k-dimensional pseudo manifold K. Because of the k-dimensional homology group Hk (K) Z of K, the three-dimensional space entity defined in this paper has connectivity. As an important topology invariant, Hk (K) can also be used as a very applicable topology consistency checking tool in the process of system design and update [17].

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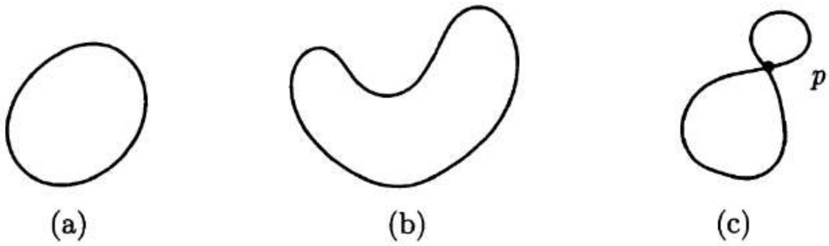

Figure 4. Three one-dimensional graphs [12].

Figure 4 shows three one-dimensional graphs, where (a) and (b) are two cycles, and (c) is an 8-word graph Although the geometric shapes of these two circles (a) and (b) are different, the graphic structure is the same, because they can be continuously deformed from one to another on the plane and coincide with it [18], without breaking or intersecting But they are obviously different from graph (c). Mathematically, the topological structures of (a) and (b) are the same, or they are homeomorphic, but different from (c). In Figure 4, (a) and (b) are simpler than (c), and they have no intersection The figure in (c) has an intersection point p.

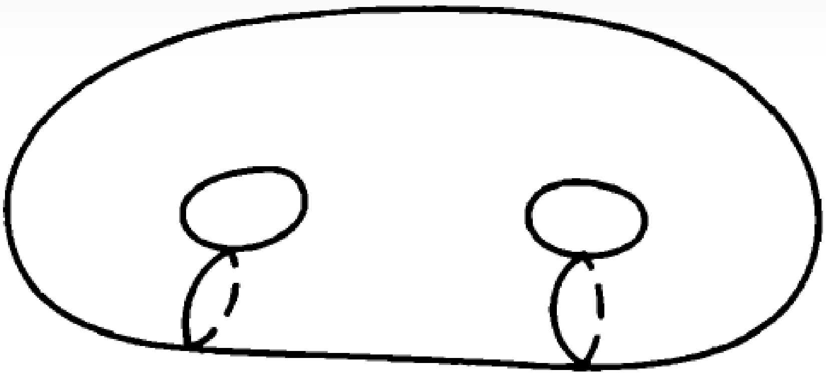

Figure 5. Two dimensional torus of one k=2 holes.

Similarly, all two-dimensional surfaces, as shown in Figure 5, are called two-dimensional manifolds.

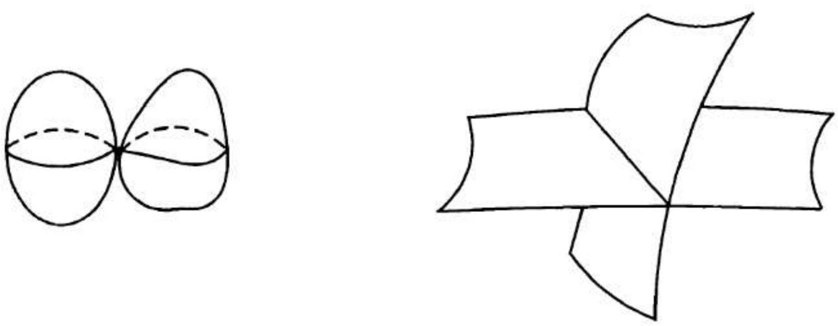

Figure 6. Intersection point of graph.

Intuitively, this n-dimensional graph without intersection points is called n-dimensional manifold in mathematics But when $n > 3$, we cannot observe the graph of this topological space, and the so-called intersection points cannot be expressed However, through careful observation, it can be found that the so-called basic feature without intersection points is that, for example, at any point p on a two-dimensional sphere S2, there is a neighborhood $U \subset S2$ of p, making U homeomorphic to an open disk D (or open sphere) of R2, as shown in Figure 6. This property does not hold for the intersection point of the graph shown in Figure 6 [19].

Define 1.1 so that M is a Hausdorff space If for any point $p \in M$, there exists a neighborhood $U \subset M$ of x in M, so that U is homeomorphic in n-dimensional Euclidean space Rn (or an open set in Rn), then M is said to be an n-dimensional manifold From the above definition, we can see that any open set in Rn is an n-dimensional manifold The manifold given in definition 1.1 has no boundary, that is, partial $M = \emptyset$ or partial $M \not\subset M$. Next, we introduce the concept of manifolds with edges, which are defined as follows. Define 1.2 so that M is a Hausdorff space. If for any point $p \in M$, there exists a neighborhood $U \subset M$, such that U is homeomorphic to an open set in $R + n$ In particular, if there is such a point $p \in M$, so that its neighborhood U is homeomorphic to $R + n$, then M is called an n-dimensional manifold with edges.

Generally speaking, the boundary of an n-dimensional manifold M with edges, partial M, must be an n-1-dimensional manifold. If the boundary of an open set A in Rn is a $n - 1$-dimensional manifold, then its closure A is a manifold with edges, otherwise A is not. In manifolds, there is a very important class called compact manifolds (also called closed manifolds). The basic characteristic of this kind of manifold is that it is bounded but has no boundary. Visualizing compact manifold images is an important step in mastering topology and geometry. Here are some simple compact manifolds. There is only one kind of one-dimensional compact manifold, S1 (circle) [20]. There are four typical types of two-dimensional compact manifold: two-dimensional spherical S2, tire surface T2 ($k =$ torus of 1 hole), Klein bottle K2 and two-dimensional real projection space P2. Because Klein bottle K2 and real projection space P2 cannot be embedded into real space R3, we cannot see them, but we can imagine and observe them in the following way. Imagine Klein bottle as shown in Figure 6, in (a) of the figure, first bend one end of the tube into the tube, and then imagine pulling this end out of the fourth dimensional space in (b), at this time, you can completely avoid intersecting with yourself, and then glue this end with the other end as shown in (c). In this way, we can observe the appearance of Klein bottle from three-dimensional space [21].

3.2. Formal description of three-dimensional topological spatial relations

In the three-dimensional topological space X, according to the degree of freedom of the spatial object, it can be divided into four kinds of spatial objects, namely, point target, linear target, area target and volume target. Point target has only position but no space expansion, so it is defined as zero dimensional target; Linear target can only expand along its line segment, so it is defined as a one-dimensional target; According to this principle, the planar target is defined as a two-dimensional target [22]; A volumetric target is defined as a three-dimensional target. Here, we only consider simple objects, that is, all objects are closed sets, and all objects are connected. By using the dimension expansion method, the dimension of the intersection of points, lines, surfaces, boundary and interior of three-dimensional space objects is taken as the description framework of spatial topological relations. The boundary of the midpoint is always empty; The boundary of the line is the two endpoints of the line, but when the line is a closed curve, the boundary of the line is empty; The boundary of a face is composed of closed curves containing extreme points of all faces; The boundary of a volume consists of a closed surface containing all the extreme points of the volume. If the boundary of three-dimensional space object A is marked LA and the interior is marked $A°$, then there is a relationship: $A° = A - LA$. Let S be a point set, and define the function din as follows:

Table 2. Formal Description.

type	condition
empty set	S=empty set
0	S does not include lines, surfaces and bodies; But at least one point
1	S does not contain faces, bodies, etc., but at least one line
2	S does not contain volume, but at least one face
3	S contains at least one person

There are two space targets A and B, and the intersection of their boundary and interior is represented

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by the following four sets:

$$S1 = LA \cap LB \quad S2 = LA \cap B^{\circ} \quad S3 = A^{\circ} \cap LB \quad S4 = A^{\circ} \cap B^{\circ}$$

The value range of $Si\ (i = 1,2,3,4)$ is: 0,1,..., din (Si). If Si has different values of Ni, then the possible topological spatial relationship between two spatial objects is $N1 \times N2 \times N3 \times N4 = N$, excluding some situations that have no practical significance, the rest can describe the topological spatial relationship between spatial objects A and B [23]. Ten topological relation sets can be defined between four spatial objects in 3D topological space. They are: point/point, point/line, point/face, point/body, line/line, line/face, line/body, face/face, face/body, and volume/body topological spatial relationships. For example, the abstract expression of a point like object is a point in its space, and its boundary is always empty. Therefore, if A is a point target, then the topological space relationship between it and other types of targets B only needs to examine the values of the set $S3 = ALB, S4 = A\,B$. Similarly, the topological spatial relationships between linear targets and linear targets, linear targets and area targets, linear targets and volume targets, area targets and area targets, area targets and volume targets, and volume targets and volume targets can also be obtained.

Klein bottle and real projection space in Figure 7, ab and a′ b′ on the upper and lower sides of the rectangle are regarded as the same [24], and then the left aa′ and the right bb′ are equivalent as shown by the arrow, that is, ab and a′ b′ are glued together, and then aa′ and b′ b are glued together in the direction of the arrow, and the resulting manifold is K2. The two-dimensional real projection space P2 is a manifold that equates (bonds) the spherical surface S2 according to the diametrical points. As shown in Figure 7, it can also be represented as a manifold obtained by equating (bonding) two sides of a disk, ab and ba, in the direction indicated by the arrow.

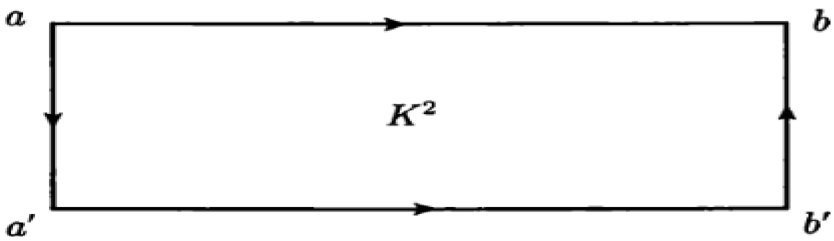

Figure 7. Rectangle.

The classification theorem of two-dimensional compact manifolds tells us that all two-dimensional compact manifolds, if not a sphere S2, can be formed by digging a disk from the tire surface T2 and the real projection space P2 and then gluing along the boundary S1, as shown in Figure 8. Klein bottle K2 is made by bonding P2 and P2, which is recorded as K2=P2 # P2=2P2. More precisely, any two-dimensional orientable compact manifold M must be homeomorphic to the torus M=kT2 of some k holes. When k=0, M is spherical, while the non orientable manifold is homeomorphic to the bonding of some n projection spaces, which is recorded as M=nP2. We will also refer to this theorem later [25].

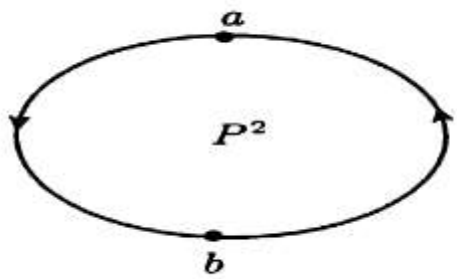

Figure 8. Disc.

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Figure 9. Two dimensional compact manifold.

3.3. Minimum set of three-dimensional topological spatial relations

The method of dimension expansion can describe a large number of topological spatial relationships existing in spatial objects, but it is difficult to remember and use these relationships. Therefore, this paper classifies all topological spatial relationships between 3D objects, and concludes five basic topological spatial relationships: adjacent, included, intersected, partially covered, and separated, thus giving a minimum set definition that conforms to user intuition and can express topological spatial relationships between all 3D objects [26]. Let the triplet<A, R, B>indicate that there is a relationship R between A and B in three-dimensional space. We call this triplet a fact, and the facts can be combined by Boolean operation intersection (Λ) or union (Λ). The formal definition of the minimum set of three-dimensional topological spatial relations is as follows.

[Definition I] Touch relation:

$$< A , \text{to uch}, B> \quad (A^\circ \cap B^\circ = \emptyset) \wedge (A \cap B \neq \emptyset)$$

[Definition II] Include (in) Relationship:

$$< A ,\text{in}, B> \quad (A^\circ \cap B^\circ = \emptyset) \wedge (A \cap B = A)$$

[Definitive Three] Intersection relationship:

$$< A , \text{cross} , B> \quad \text{din}(A^\circ \cap B^\circ) < (\max (\text{di n}(A^\circ) \; \text{din}(B^\circ)) \wedge (A \cap B \neq A) \wedge (A \cap B \neq B)$$

[Definition IV] Partial overlay relationship:

$$< A, \text{overlap}, B > \quad (\text{din}(A^\circ \cap B^\circ) = \text{di n}(A^\circ) = \text{din}(B^\circ)) \wedge (A \cap B \neq A) \wedge (A \cap B \neq B)$$

[Definition V]Disjoint relation:

$$< A , \text{dis joi nt}, B> \quad A \cap B = \emptyset$$

Table 3. Minimum Set of 3D Topological Spatial Relations.

category	relationship
[Definition 1]	Touch relation
[Definition 2]	Include (in) Relationship
[Definition 3]	Intersection relationship
[Definition 4]	Partial overlay relationship
[Definition 5]	Disjoint relation

Figure 10 shows some illustrations of the five topological spatial relationships between linear target A and volumetric target B. The minimum set of three-dimensional topological spatial relations defined above is mutually exclusive and complete. There are two spatial objects A and B, and there is a relationship R between them. If (A, R, B) is true, then it is not true for all Ri ≠ R, <A, Ri, B>, and no other topological relations beyond the five relations defined above exist. That is, the topological relationship between two 3D spatial objects can only be one of the five topological spatial relationships defined above. This conclusion can be proved in the following ways. Any two spatial objects A and B are either separated from each other or related to each other. Their relationship can be represented by the topological spatial relationship decision tree between three-dimensional objects as shown in figure 10. In Figure 11, each internal node represents a Boolean operation. If its value is true, it goes to the left subtree, otherwise it goes to the right subtree. Repeat this process until the leaf node represents one of the five basic topological relationships.

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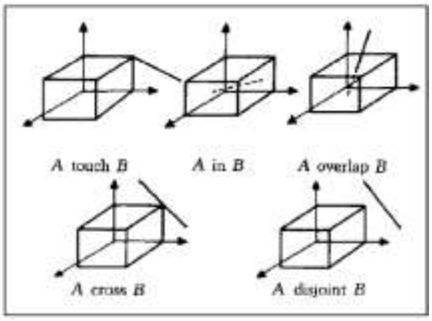

Figure 10. Topological relationship between linear target A and volumetric target B [12].

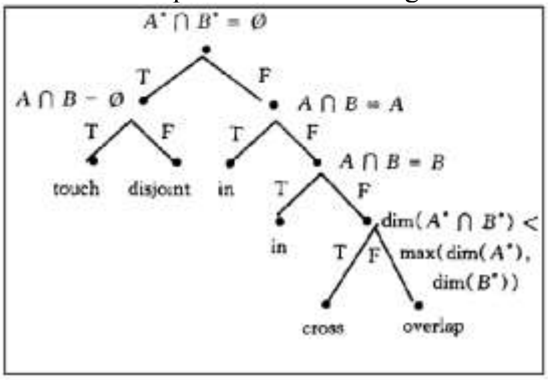

Figure 11. Decision tree of topological spatial relationship between 3D objects [12].

From the topological relation decision tree, it can be seen that for any given two spatial objects, there can be no two different topological spatial relations, because there is only one path for operation in the topological relation decision tree, and no other relations beyond the five relations defined above exist, because each internal node has two branches, so that each Boolean operation value has its corresponding path; Each leaf node exactly corresponds to each topological space relationship described by the expansion of the dimension of the five defined topological space relationships, and can be represented by the logical intersection of four conditional expressions reflecting the boundary and internal intersection between two objects, which can be recorded as:

$$T1(LA \cap LB \cap) \wedge T2(LA \cap B^{\circ}) \wedge T3(A^{\circ} \cap LB) \wedge T4(A^{\circ} \cap B^{\circ})$$

Each term Ti corresponds to a logical expression Pi, which is composed of target and boundary and five defined topological spatial relationships. Replace Ti with each Pi to obtain:

$$P1 \wedge P2 \wedge P3 \wedge P4$$

Therefore, the five topological spatial relations defined above can fully express all the relations described by the dimension expansion method.

4. Conclusion

In conclusion, this paper clarifies the necessity of studying three-dimensional spatial data models, analyzes the research progress and existing problems of topological spatial relationship description methods, and proposes a complete and formal description framework of three-dimensional topological spatial relations based on point set topological theory and dimension expansion method. On this basis, the topological spatial relations existing in three-dimensional spatial objects are classified, and five fundamental topological spatial relations are defined. The mutual exclusivity and completeness of the minimal set of 3D topological space relations are proved. Based on point set topology and k-dimensional pseudo-manifold, this paper proposes the formal description of three-dimensional topological properties and spatial relations between spatial entities. The simplicial homology is applied to manifolds, the topological properties of three-dimensional space entities are revealed, and the formal description method of three-dimensional space entities is given through the geometric structural relationship between simplex and simplicial k-complex.

Furthermore, this kind of research will help to design effective data management, spatial query and spatial analysis methods in GIS. On this basis, the author of this paper establishes a 3D spatial data model based on the combination of boundary representation (BR) and geometric voxel construction (CSG), and realizes a quick query of 3D topological spatial relations in this data model, which provides a basis for 3D spatial analysis.

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