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Algorithmic applications and principles of superconducting quantum computers

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Abstract. Nowadays, with the continuous progress of science and technology, the role of quantum computers has received more attention and gradually become a part of national strategies. Many countries, such as the United States, China, and Japan, have invested a lot of resources in research and development, striving to make breakthroughs and take the lead in this field, in order to meet the coming of the future computing era. Superconducting quantum computers are the most widely used computers at the moment. Based on the introduction of Josephson junction, the physical principle of superconducting quantum computers, this paper analyses the advantages and disadvantages of multiple quantum bits as well as the algorithms of various quantum gates in superconducting quantum computers. Moreover, the author also looks forward to the development of the related technologies of superconducting quantum computers. Finally, corresponding suggestions are put forward for some current problems.

Keywords: Superconducting Quantum Computer, Josephson Junction, Quantum Bit, Quantum Gate.

1. Introduction

Since Bohr and other scientists proposed quantum mechanics in the last century, quantum computers were finally born after continuous efforts and research. The research of quantum computers in the field of science and technology is of great significance. Therefore, various countries have increased their research and investment in quantum computers to compete for the competitive advantage of science and technology in the field of quantum computing. There are many types of quantum computers, which are roughly divided into two categories: ion-trap quantum computers [1] and superconducting quantum computers. These different types of quantum computers have their own characteristics and advantages, and each of them plays an important role in research and application. Superconducting quantum computers are one of the more technologically mature ones. The principle of Joseph junction is one of the basic principles of superconducting quantum computers. This principle is based on the low-temperature property of superconducting materials, and by cooling the superconducting materials below a critical temperature, zero impedance transmission of current can be achieved, thus avoiding energy loss. In the design of superconducting quantum computers, quantum bits are one of the key elements. A quantum bit is the smallest unit of information in quantum computing, similar to a bit in classical computing. Unlike bits in classical computers that can only represent two states, 0 or 1, quantum bits can be in a superposition of 0 and 1 at the same time, which is unique to quantum

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computing. In addition to the choice of quantum bits, quantum gates are also an important component in superconducting quantum computers. Quantum gates are the basic logic gates used to implement interactions and operations between quantum bits. Common quantum gates include Hadamard gates, CNOT gates, etc. In this paper, by dissecting and studying the various quantum gates, it is possible to conclude which quantum bits and quantum gates are most likely to be realised, thus making a valid prediction for a superconducting quantum computer.

2. Josephson junction

Josephson junction plays an integral role in superconducting quantum computers [2]. It was proposed by the British physicist Ryan Josephson, for which he was awarded the Nobel Prize in Physics in 1973. Its design principle is based on the quantum tunnelling effect. Quantum tunnelling is a quantum mechanical phenomenon that allows microscopic particles to pass through energy barriers that classical physics considers impossible to cross. In classical physics, it is believed that there must be enough energy to overcome the potential barrier in order to cross over. However, quantum tunnelling shows that even without sufficient energy, particles can penetrate potential barriers and appear in places people would not expect. The principle of quantum tunnelling can be described by the wave function in quantum mechanics. The wave function is a mathematical function that describes the behaviour of a particle and contains the probability distribution of the particle in space. When a particle encounters a potential barrier, the wave function decays, but there is still a certain probability that the particle will cross the barrier. This probability is related to the height and width of the potential barrier, as well as the energy of the particle. Here, according to Schrödinger, the penetration rate and reflectivity can be calculated. Since there are a large number of calculations, they will not be all listed in this paper. The Josephson junction is a special structure which utilises the quantum tunnelling effect. The structure consists of two superconductors between which sandwiched a layer of non-superconducting insulating material that is roughly only 3nm thick. Electrons in the superconductors are able to move in pairs, forming Cooper pairs. The Cooper pairs, in turn, circumvent energy loss through lattice vibrations in the superconductor. However, ordinary electrons in a superconductor lose energy due to the presence of electrical resistance. The insulating layer of the Josephson junction is thin enough so that Cooper pairs can pass through the insulating layer through quantum tunnelling effects without suffering energy loss. When electrons flow through a Josephson junction, Cooper pairs can pass through the insulating layer through quantum tunnelling. When they encounter an insulating material, again due to the quantum tunnelling effect, they can pass through the insulating layer, thus forming a new pair of electrons. This phenomenon is known in quantum mechanics as "electron pair conversion". In this way, Josephson junctions can actually change the number of electron pairs. This is important for quantum computing and quantum information processing. Unlike conventional LC oscillator circuits, the energy levels and Hamiltonian quantities of a conventional LC oscillator circuit satisfy the following equation [3].

$$E_n = \left(n + \frac{1}{2}\right) hm \tag{1}$$

$$H = 4E_C n^2 + \frac{1}{2} E_L \varphi^2 \tag{2}$$

Note that h is the $\frac{\text{Planck's constant}}{2\pi}$, m is the mass, E_C is the energy of the capacitance, m is the energy of the stem, and h is the Hamiltonian. E_l is the energy of the electric stem, and H is the Hamiltonian. Its energy levels are in the form of a ladder with equal energy between every two levels. It cannot be better controlled, while the Josephson junction has different energy levels and its main voltage and current have to satisfy the following equation [3]:

$$I = I_c \sin\varphi \tag{3}$$

$$V = \frac{h}{2e} \frac{\partial \varphi}{\partial t} \tag{4}$$

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The equation for the current can be introduced by the equation for electromagnetism, while the voltage is introduced by Schrödinger's equation, and the integration yields several equations for the electrostatic energy [3].

$$E = \int_0^t \frac{h}{2e} I_C \sin\varphi \frac{\partial\varphi}{\partial t} = -E_j \cos\varphi \quad (5)$$

E_J is the Josephson junction energy, and ultimately its equation with the Hamiltonian quantity can be written as follows [3]:

$$H = 4E_C n^2 - E_j \cos\varphi \quad (6)$$

From the equations above, it can be known that the energy levels are not stepped and there is a large gap between each of the two energy levels so that the two states can be better controlled. The core components of the superconducting quantum interferometer are two Josephson junctions. Josephson junctions are structures made of alternating superconductors and insulators, which have superconductivity and quantum interference effects. In a superconducting quantum interferometer, these two Josephson junctions are connected into a ring, forming a closed loop. When a superconducting quantum computer performs calculations, the interaction between the quantum bits can be controlled by applying different currents to the Josephson junctions. This interaction is realised by quantum interference effects, i.e. superposition states and mutual interference between quantum bits. By adjusting the magnitude and direction of the current, the quantum states in the interference loop can be changed, thus enabling the control and processing of the quantum bits.

3. Types of superconducting quantum bits

Conventional superconducting quantum bits are classified into three types: the charge qubit, flux qubit, and phase qubit. Superconducting quantum computation is affected by various noises, which leads to an increase in the error rate of the computation. Thermal noise is a major source of noise in superconducting quantum computation. Thermal noise is caused by temperature inhomogeneity, which causes the interaction between quantum bits to become complicated. When the temperature increases, the coupling effect between quantum bits is enhanced, which, in turn, leads to a decrease in the accuracy of the calculation. In addition to thermal noise, superconducting quantum computing is also challenged by internal noise. Internal noise is caused by various undesirable factors in superconducting circuits, such as current fluctuations and energy level crossovers. These factors cause the interactions between quantum bits to become unpredictable, thus affecting the accuracy of the computation. Therefore more quantum bits are designed such as transmon, Xmon, Gmon, 3Dtransmon, C-shunt, Fluxonium, 0-π, and other super quantum bits.

3.1. Conventional superconducting quantum bits

In a charge quantum bit, the inductive part is removed and the bit is regulated by a voltage modulated by an external field [4]. The structure of a capacitor and a Josephson junction form an island, also known as a Cooper-Pair Box, whose energy mainly satisfies the ($\frac{E_j}{E_C} \leq 1$).

The flux qubit consists of an inductor and a Josephson junction. The quantum bit and the superconducting currents of the encode in two different directions in the circuit correspond to the lowest energy levels of the two sides of the potential well. The jump between the two states is accomplished by lowering the height of the intermediate potential barrier. Its energy mainly satisfies ($1 < \frac{E_j}{E_c} \leq 100$).

The phase qubit changes the potential well in the form of a cosine function of charge quantum bits by DC biasing the junction. This enables only a few energy levels to exist in a given potential well and encodes the lowest two energy levels as quantum bits. Their energies mainly satisfy ($100 < \frac{E_j}{E_C}$).

3.2. Transmon
Transmon was first proposed by Koch et al. in 2007 [5]. In conventional charge-volume bits, the charge dispersion decreases proportionally with $\frac{E_J}{E_C}$, while the anharmonicity decreases with the slow power-law of $\frac{E_J}{E_C}$. Transmon, on the other hand, works by connecting a large mutually exclusive capacitor in parallel with a Josephson junction, thereby appropriately increasing $\frac{E_J}{E_C}$ and greatly decreasing the sensitivity of the system to the charge while maintaining sufficient anharmonicity.

3.3. Xmon
Xmon is a new superconducting quantum bit which can be regarded as an improved version of Transmon [6]. It is an improved and optimised version of Transmon. Xmon has a very simple structure, consisting of a crossed capacitor. Unlike the traditional Transmon, the Xmon quantum bits are coupled to a common transmission line through a resonant cavity. This structure is designed to give Xmon greater direct connectivity. In Xmon, each quantum bit has two independent control lines, the XY control line and the Z control line. These two control lines can perform rotational operations on the quantum state in the X, Y, and Z directions, thus realising the control of the quantum bits. At the same time, the quantum bits of Xmon can be directly coupled with each other through capacitors. This direct coupling design gives Xmon the property of fast control while maintaining long coherence.

3.4. Three-dimensional Transmon
3D Transmon has several advantages over Transmon [7]. Firstly, it has a larger die volume and is therefore less sensitive to surface medium loss. This means that 3D Transmon has better performance in quantum computing. Second, this structure provides a better electromagnetic environment for quantum bits. This suppresses the decoherence of quantum bits, allowing the coherence to last longer and maintaining the coupling to the control information.

3.5. C-shunt
The C-shunt can be seen as an improved version of the flux, with improvements in charge noise [8]. It appears as a flat spectrum in the image of the energy, with a longer coherence time, effectively improving the decoherence time. The sensitivity of the C-shunt to dielectric loss has been reduced, and the C-shunt has also shown important improvements in the reduction of dielectric loss. This implies that the C-shunt in superconducting quantum computers can be further improved in terms of performance and utility by optimising the design and technology for more efficient energy transfer.

3.6. 0-π
0-π is one of the newest quantum bits with great potential [9]. 0-π's two ground state wave functions are highly localised in their respective potential wells and are not disconnected from each other. This means that in 0-π, there is a strong interaction between the two ground state wave functions and there is no chaos as in normal quantum systems. This localisation gives 0-π excellent fidelity and stability. On this basis, 0-π can be used to realise applications such as quantum communication, quantum key distribution, and quantum computation.

4. Superconducting quantum bit gates
In superconducting quantum bits, apart from the mentioned Hadamard gates, Pauli X gates, Pauli Z gates, Pauli Y gates, and double quantum bit gates, there are some other gates and operational gates like CNOT gates [10], Toffoli gates [11], Grover gates [12] and so on. Among them, Hadamard gates are used to implement interconversion between super quantum bits to achieve interconversion between quantum bits. Its function is to transform a quantum bit from a classical bit (0 or 1) to a superposition state, i.e., to place a quantum bit in a 0 and 1 superposition state. It can convert the ground state $|0\rangle$ to ($|0\rangle+|1\rangle/\sqrt{2}$) and the ground state $|1\rangle$ to ($|0\rangle-|1\rangle/\sqrt{2}$) whose matrix is $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$.

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Pauli X-gate, Pauli Z-gate, and Pauli y-gate are also classical quantum bit gates and belong to the category of Pauli gates, which are obtained by rotating around the x or y or z axis. Measurement of superconducting quantum bits using the Pauli X-gate yields superconducting quantum bit x-state and z-state measurements with the matrix of $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Measurement of superconducting quantum bits using the Pauli Z-gate also yields a measurement of the Z-state of superconducting quantum bits with a matrix of $\begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$. The Pauli Y-gate can be used to control the interactions between superconducting quantum bits to perform certain steps of a quantum algorithm with a matrix of $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

A double-qubit gate is used to run two quantum bits. It can be used for interactions and transformations between quantum bits, which include quantum entanglement, quantum state exchange, and superposition. A range of quantum computing operations such as quantum state swapping, superposition, measurement, and quantum entanglement can be implemented using a two-qubit gate. These operations are important for quantum computing and quantum information processing, and they also provide the basis for the implementation of more complex quantum algorithms and protocols. The double-qubit gate fidelity of superconducting quantum bits is approaching 98%.

A CNOT gate consists of a superconducting quantum bit and an anti-superconducting quantum bit. The CNOT gate performs an inverse operation on the target bit when the control bit is 1; if the control bit is 0, the target bit remains unchanged. Simply put, the CNOT gate function is to control the state of the bit, thus determining the change of the target bit.

Toffoli gates are based on CNOT gates with the addition of quantum bits.

Grover gates belong to the category of parallel gates, which consist of several gates that enable parallel operations between several quantum bits. Search is an important and time-consuming task in the process of quantum computing. Conventional search algorithms generally examine each possible solution one by one, which can be inefficient when searching large-scale data. This is changed by the existence of Grover gates, which can be applied to speed up the search, especially for large-scale data searches. It operates in parallel to find the target solution in constant time. This parallel operation feature turns Grover gates into a powerful tool and significantly improves the efficiency of the search. In addition to accelerating the search, Grover gates can also be used for parallel measurements and other parallel operations. Simultaneous measurements of multiple quantum bits can yield more information in a shorter period of time. This parallel measurement capability can be useful for a number of applications, especially when large amounts of data need to be processed efficiently.

The combination of different gates enables different operations with different relations between quantum bits. For example, the sum relation between quantum bits can be realised by combining 2 CNOT gates with 1 Hadamard gate. Firstly, one bit is converted into a bit in the superposition state by a Hadamard gate. Then, a CNOT operation is performed with the first bit using the other bit as the control bit. Finally, the first bit is restored to its original state again by a Hadamard gate. This allows the transfer of the original bit to the final state and also ensures that no additional storage information is required in subsequent operations. In practice, CNOT is generally simpler to implement than Hadamard. However, there are times when the results achieved using CNOT gates are less than desirable. It is therefore necessary to explore further ways to improve this performance.

5. Conclusion

In conclusion, most of the modern superconducting quantum computers are based on the Josephson junction principle, and for all kinds of quantum bits, 0-π is least affected by noise, and the functions of different quantum gates are not the same. Among quantum computer technologies, superconducting quantum computer technology is more mature, and many countries have taken the lead in realising it. However, this technology also faces a series of challenges and problems. One of the main problems is to cool the superconductor to a low temperature of -273 degrees to achieve its normal operation. This is due to the fact that at extremely low temperatures, superconductors will show the strange phenomenon of unimpeded flow of electric current. This superconducting phenomenon gives quantum computing a

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stable and reliable computing environment. Superconducting quantum computers typically use dilution chillers to achieve this environment [13]. There are of course problems such as short coherence times, and the various superconducting quantum bits mentioned in this paper are designed to address such problems. Secondly, the number of quantum bits and computational accuracy of superconducting quantum computers is still limited, which restricts its feasibility in practice. In addition, some key technologies in superconducting quantum computers such as error correction and quantum entanglement have yet to be thoroughly researched and developed. Despite these challenges, with the continuous development of the technology, it is believed that superconducting quantum computers will play an important role in the future and promote the process of scientific research and technological innovation.

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