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Analysis of mechanical principle of banana ball

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Abstract. The banana ball is a famous way to score goals in football matches. In the flight, the ball bypasses the defensive wall, giving the goalkeeper the illusion that the ball is far away from the goal, and then the ball will suddenly change direction. In order to improve the scoring probability and make training more efficient and scientific, in this article, the mechanics principle of banana ball is analyzed. A complete and comprehensive analysis of the ball motion from kick-out to flight in the air is presented. The analysis of flight considers the magnitudes of air resistance, Magnus force and Bernoulli's principle. The complete equation of motion is established. The numerical simulation by Matlab is used to simulate flight path. The flight curve of banana ball is successfully simulated. The result is consistent with real phenomenon. This article is useful for athletes and football lovers with interest in banana ball practice.

Keywords: Banana ball, Bernoulli’s principle, Magnus force

1. Introduction

Scoring directly from the free kick is a mean of goaling in football matches. With the continuous development of modern football techniques and tactics in recent years, the goal from free kick is more and more common. Banana ball is a famous one among free kick methods. It is difficult to defend. The ball bypasses the defensive wall, giving the goalkeeper the illusion that the ball is far away from the goal, and then suddenly changes direction in flight. The direction of the goal contracts, forming a banana shaped arc, hence this phenomenon is named banana ball[1]. Through the analysis of its principle, theoretical basis for the free kick break can be revealed, and the quality and efficiency of training will be improved. As a consequence, matches will be more enjoyable and scientific, and the development of football will be promoted. In the previous researches, T.Asai[2] analyzed effects that are caused by leg swinging to initial speed of the ball. Zhao[3] analyzes forces during flight and establishes equations of motion. Che[4] visualizes the numerical solution of the equation. However, none of these research makes analysis on the whole process of the banana ball. In this article, a complete and comprehensive analysis of the ball motion from kick-out to flight in the air is presented. The analysis of flight considers the magnitudes of the air resistance and Magnus force and Bernoulli's principle involved in the flight. Based on these, the equation of motion is established. The equation of motion is numerically solved by Matlab, and flight simulation is carried out. The flight curve of banana ball is successfully simulated. The numerical result is consistent with real phenomenon. This article is useful for athletes and football lovers with interest in banana ball practice.

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2. Analysis of forces during flight

2.1. The overall analysis of force

The banana ball is formed because of a pressure difference between the sides of the ball as it spins[5,6]. Therefore, the air resistance and the rotation of the ball are needed to be concerned. In order to simplify the analysis, the overall force can be divided into three parts: gravity, air resistance, and Magnus force. The forces are shown in the Figure 1.

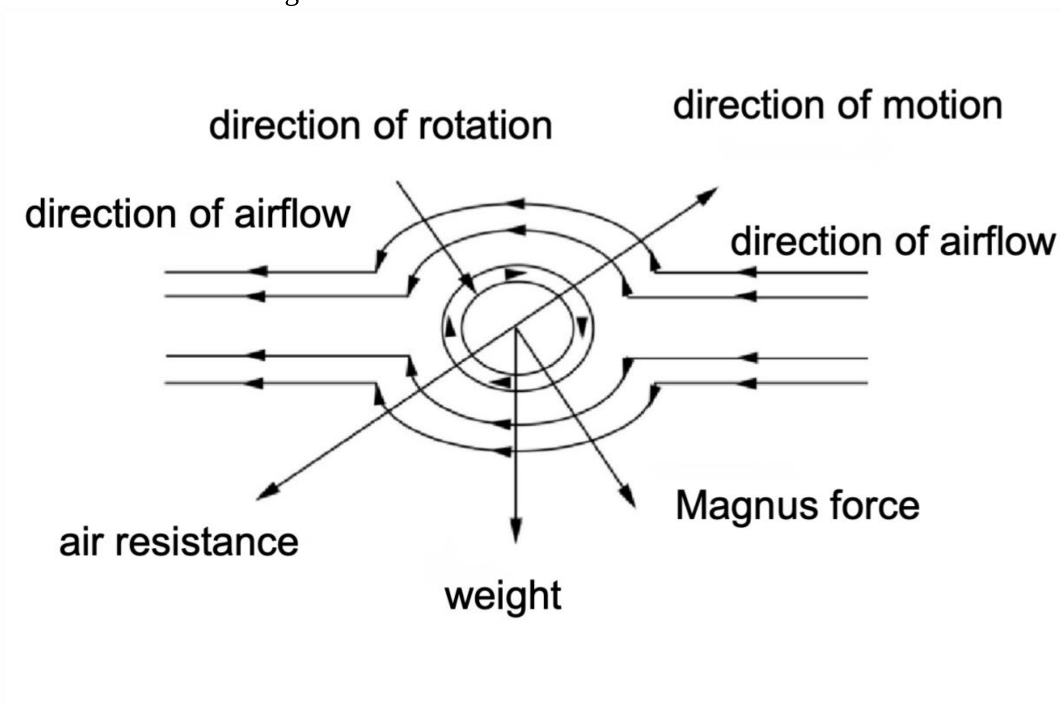

Figure 1. Forces loaded on banana ball[3]

2.2. Bernoulli principle

In order to analyze the Magnus force and air resistance, Bernoulli principle is introduced. The essence of the Bernoulli principle is that the sum of the kinetic, pressure and potential energy of an ideal fluid that is incompressible with negligible viscosity is a constant. It can be written as:

$$P_1 + \frac{1}{2}\rho {v_1}^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho {v_2}^2 + \rho g h_2 = \mathrm{C} \quad (1)$$

P is the pressure, v is the velocity, ρ is the density of air, g is the gravitational acceleration, h is the height, and C is a constant.

2.3. Magnus force

As shown in Figure 2, since the ball is rotating during flight, the velocity of air flow of Side A is greater than that of the Side B. According to the inference of Bernoulli's principle, the pressure is low where the velocity is high in a fluid of equal height. Therefore, the two sides of the ball will produce pressure difference. It is because of the pressure difference between the two sides that the Magnus force is created according to the Magnus effect[7,8]. And it is the Magnus force that makes the trajectory of the football form an arc shape like banana.

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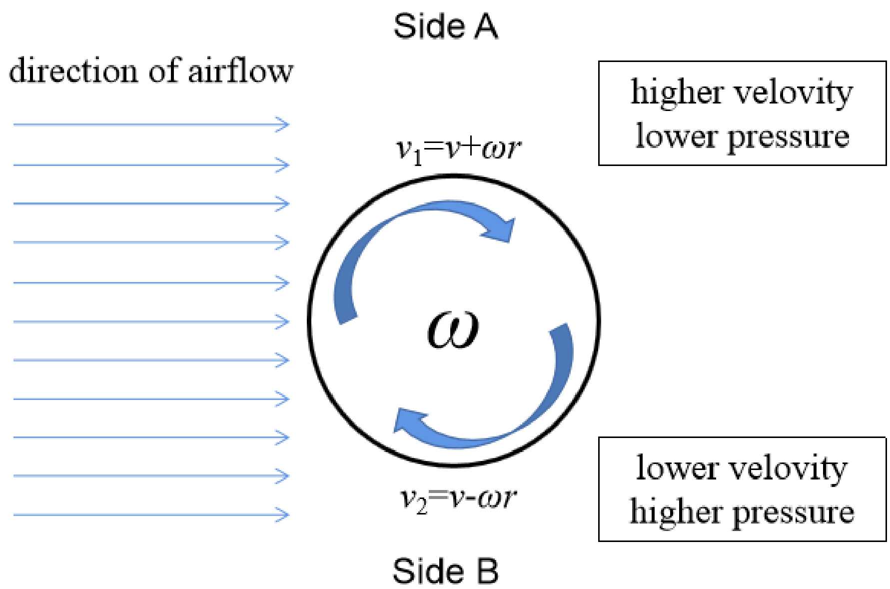

Figure 2. Schematic diagram of airflow velocity around banana football

To calculate the Magnus force, the velocity on both sides can be expressed as:

$$v_1 = v + \omega r \tag{2}$$

$$v_2 = v - \omega r \tag{3}$$

v is the velocity of the ball, ω is the angular velocity of the ball, r is the radius of the ball.

According to equation (1), since the difference in altitude between the two sides of the banana ball is so small that it can be ignored. The pressure difference between the two sides can be written as:

$$P_2 - P_1 = \frac{1}{2}\rho v_1{}^2 - \frac{1}{2}\rho v_2{}^2 = 2\rho v \omega r \tag{4}$$

Thus, the Magnus force caused by the pressure difference is calculated:

$$F_m = 2\rho v \omega r \cdot \pi r^2 \tag{5}$$

As the surface is not perfectly circular, the constant C_l is introduced to express the relationship between Magnus force and related parameters:

$$F_m = \frac{1}{2} C_l S \tag{6}$$

S is the area of the ball.

2.4. Air Resistance

The air resistance can also be calculated by calculating the pressure difference[9]. The sphere is divided into two parts to study the pressure difference, as shown in the Figure 3:

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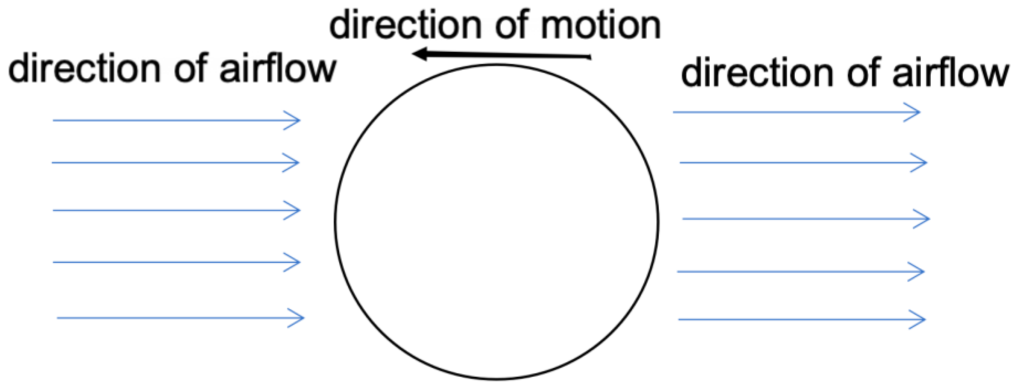

Figure 3. Velocity images for two positions

According to equation (1), velocity is expressed by the relative velocity on both sides of the ball, $v_1=v$ and $v_2=0$. So the pressure difference between the two sides is written as:

$$\Delta P = \frac{1}{2}\rho v^2 \tag{7}$$

Therefore, the pressure difference in the ideal state, namely the resistance, can be written

$$F_r = \frac{1}{2}S\rho v^2 \tag{8}$$

As the ball is not a regular flat object, the final force will be affected by the surface gap of the ball, materials and other factors, so the drag force parameter C_d is introduced:

$$\mathrm{F_r} = \frac{1}{2}C_d \mathrm{S}\rho v^2 \tag{9}$$

3. Mechanics of kicking a ball

According to Bernoulli's principle, the rotation of the ball produces the formation of banana ball. Therefore, the total torque cannot be zero, and the initial force cannot pass through the center of gravity of the ball. The kicking force can be divided into the normal direction and the tangential direction. The tangential direction will cause the ball to spin, while the normal direction will determine how far the ball will travel.

To verify how the velocity and angular velocity of a sphere are affected by the initial force of a football, a FIFA official ball is tracked and analyzed by a high-speed camera with 4500 frames per second to explore how the initial velocity of the ball is affected by the offset distance of the kicking force [10].

As kicking involves too complex muscle structure in legs and feet, the content related to biomechanics will not be elaborated here. The following analysis will simplify force into a contact force ***F*** directly in contact with the ball.

First, according to the impulse momentum theorem, the initial velocity of the ball can be written as:

$$\boldsymbol{v}=\frac{\boldsymbol{I}}{m} \tag{10}$$

I is the impulse of the kicking and *m* is the mass of the ball.

According to the collected data, the above expression conforms the actual situation.

Table 1. Data of initial velocities and impulse of kicking [10]

	A	B	C	D	E	F	Mean
velocity(m/s)	25.17	25.21	24.46	25.96	26.98	25.21	25.44
Impulse (N·s)	10.93	10.96	10.63	11.28	11.59	10.96	11.06

The next step is to analyze how the position of the kicking affects the initial linear velocity of the ball: L is the distance of the force relative to the center of the circle. L is used to reflect the kicking position, as shown in Figure 4.
According to the conservation of momentum, the initial velocity can be expressed as:

$$v = \frac{m_1}{m_2} v_0 \cos(\sin^{-1}\frac{L}{r}) \tag{11}$$

m_1 is the simplified mass of the leg and foot, m_2 is the mass of the ball, v_0 is the velocity of the simplified kicking and r is the radius of the ball.

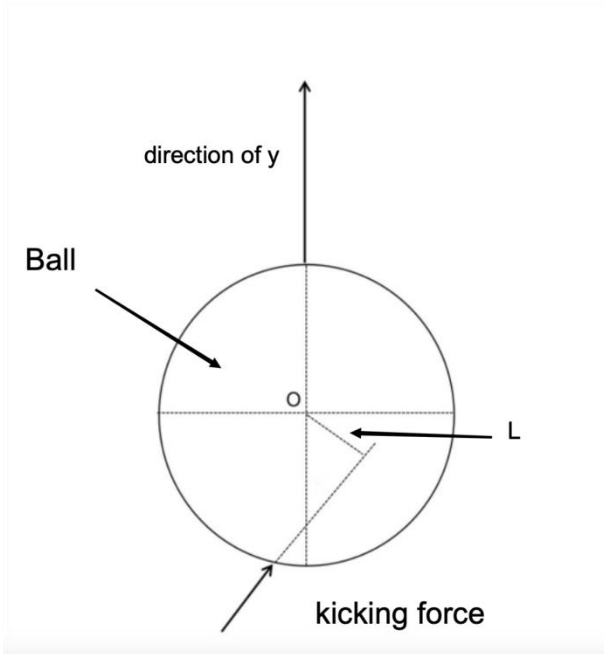

Figure 4. Force image of ball during injection

By studying the data from the experiment, it can be found that the relationship between velocity and offset distance is indeed consistent with the equation (11). It can also be found from the expression and data that when the offset distance is less than the radius, the velocity decreases with the increase of the offset distance.

Table 2. Data of offset distances of kicking force and initial velocities of ball[10]

Case	Offset distance (m)	Ball velocity (m/s)
1	-0.10	19.0
2	-0.08	20.5
3	-0.06	22.9
4	-0.04	23.5
5	-0.02	25.5
6	0.0	26.0
7	0.02	25.9
8	0.04	23.1
9	0.06	20.7
10	0.08	18.5
11	0.10	15.1

The same method of momentum conservation is adopted to regard the foot and the ball as a system free from external forces, and the linear momentum of the foot is converted into the rotational momentum

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of the ball:

$$m_1 v_0 \frac{L}{r} = \frac{2}{3} m_2 r^2 \omega \tag{12}$$

The initial angular velocity is expressed as:

$$\omega = \frac{3m_1}{2m_2} v_0 \frac{L}{r^3} \tag{13}$$

4. Equation of motion

4.1. Establishment of equation of motion

Since the directions of three forces are relatively complicated during flight, it is necessary to establish the motion equation of the three directions *xyz* through Newton's second Law to analyze how the position of the ball is affected by the relevant parameters.

The forms of the three forces into vector forms are organized[3]:

$$F_g = mg \tag{14}$$

$$F_r = -\frac{1}{2} C_d \rho S v^2 \frac{\boldsymbol{v}}{v} \tag{15}$$

$$F_m = \frac{1}{2} C_l S \rho v \omega r \frac{\boldsymbol{\omega} \times \boldsymbol{v}}{|\boldsymbol{\omega} \times \boldsymbol{v}|} \tag{16}$$

Equation (16) is converted to the form like equation(15)

$$F_m = \frac{1}{2} C_l \rho S v^2 \frac{\boldsymbol{\omega} \times \boldsymbol{v}}{|\boldsymbol{\omega} \times \boldsymbol{v}|} \tag{17}$$

$\boldsymbol{\omega} \times \boldsymbol{v}$ will be transformed into three directions:

$$\boldsymbol{\omega} \times \boldsymbol{v} = \begin{vmatrix} i & j & k \\ \omega_x & \omega_y & \omega_z \\ v_x & v_y & v_z \end{vmatrix} = (\omega_y v_z - \omega_z v_y)\mathbf{i} + (\omega_z v_x - \omega_x v_z)\mathbf{j} + (\omega_x v_y - \omega_y v_x)\mathbf{k} \tag{18}$$

A second order ordinary differential equation is established for the forces in three directions:

$$\begin{gathered} \frac{d^2x}{dt^2} = -\frac{1}{2m} C_d \rho S v^2 \frac{v v_x}{v|v_x|^2} v_x + \frac{1}{2m} C_l \rho S v^2 n_x \\ \frac{d^2y}{dt^2} = -\frac{1}{2m} C_d \rho S v^2 \frac{v v_y}{v|v_y|^2} v_y + \frac{1}{2m} C_l \rho S v^2 n_y \\ \frac{d^2z}{dt^2} = -\mathrm{g} - \frac{1}{2m} C_d \rho S v^2 \frac{v v_z}{v|v_z|^2} v_y + \frac{1}{2m} C_1 \rho S v^2 n_z \end{gathered} \tag{19}$$

and:

$$\begin{cases} n_x = \dfrac{\omega_y v_x - \omega_z v_y}{\sqrt{(\omega_y v_x - \omega_z v_y)^2 + (\omega_z v_x - \omega_x v_z)^2 + (\omega_x v_y - \omega_y v_z)^2}} \\ n_y = \dfrac{\omega_z v_x - \omega_x v_z}{\sqrt{(\omega_y v_x - \omega_z v_y)^2 + (\omega_z v_x - \omega_x v_z)^2 + (\omega_x v_y - \omega_y v_z)^2}} \\ n_z = \dfrac{\omega_x v_y - \omega_y v_z}{\sqrt{(\omega_y v_x - \omega_z v_y)^2 + (\omega_z v_x - \omega_x v_z)^2 + (\omega_x v_y - \omega_y v_z)^2}} \end{cases} \tag{20}$$

Since the second-order ordinary differential equation cannot be solved directly, the numerical solution using the Runge-Kutta algorithm in Matlab is used to solve this problem.

4.2. Numerical simulation

In this article, parameters are set as m=0.445kg, S=0.03m^2, $\rho = 1.29\mathrm{kg} \cdot \mathrm{m}^{-3}$, C_d=0.35, C_l=0.7. The simulated flight curve is shown in Figure 5.

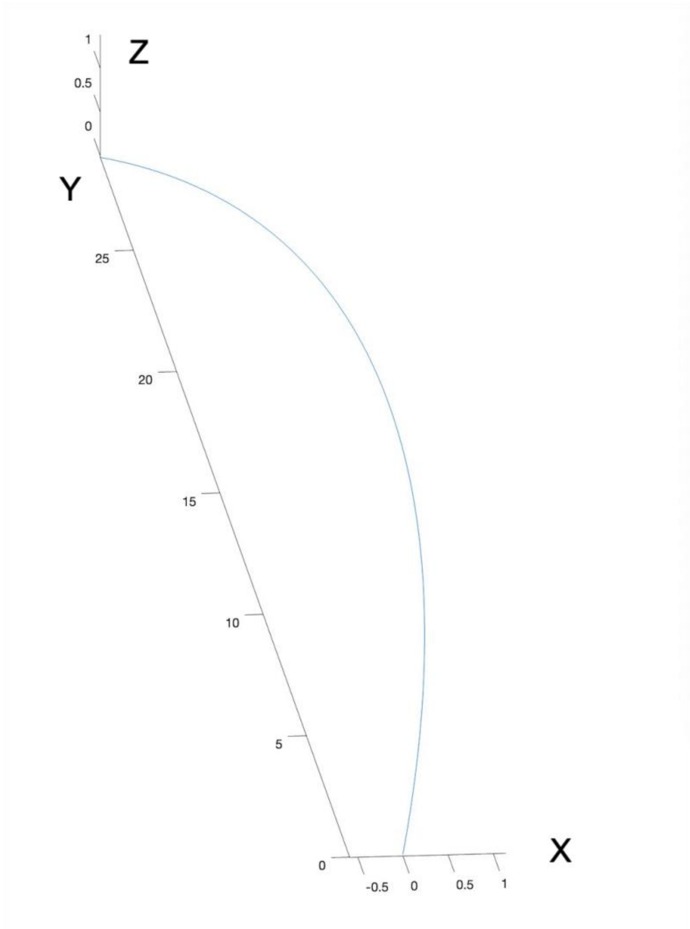

Figure 5. Simulated flight curve of banana ball

Y is the direction of goal; *X* is the direction of excursion and *Z* is the vertical direction. The initial ball is at (0,0,0) and the end point is at (-0.6, 29, 0). The flight curve shows a shape of arc, just like a banana. The maximum offset distance is 1.2m and the highest point is at 1.4m. The offset distance increases at first and then decreases in the *X* direction. All of these are consistent with real phenomenon, verifying the correctness of the simulation method.

5. Conclusion

Banana ball is a famous way to score goals. The ball can bypass the defensive wall and make a goal. In this paper, in order to analyze the principle of banana ball, a complete and comprehensive analysis of the ball motion from kick-out to flight in the air is presented. The analysis of flight considers the magnitudes of air resistance, Magnus force and Bernoulli's principle. Based on these, the equation of motion is established. The equation of motion is numerically solved by Matlab. The flight simulation is carried out, and the numerical result is consistent with real phenomenon. Therefore, the method in this paper can be used to guide the practice of banana ball. This article may offer a reference for the improvement of free-kick training quality and efficiency.

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