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Analysis of application of FFT in quantum field

Zhaoyang Xu

Department of Maths, China University of Geosciences, Wuhan, 430074, China

xp57@scarletmail.rutgers.edu

Abstract. The algorithm concept of fast Fourier transform, Heisenberg uncertainty, and the combination of the two are expounded. Fast Fourier Transform (FFT) is not a new discrete Fourier Transform, but a fast algorithm for Discrete Fourier Transform (DFT). For a long time, the DFT was not really used because of the large amount of computation, even if it was used by computers, it was difficult to deal with problems in real time. It was not until when Cooley and Key first proposed a fast algorithm for DFT operations, and later, when fast algorithms by G. Sullivan and. Key appeared in succession, that fundamental changes took place. People began to realize some inherent laws of DFT operation, and thus quickly developed and improved a set of high-speed and effective operation methods, which is now commonly known as the fast Fourier transform FF11 algorithm. Specifically, the position of a particle and its momentum cannot be determined simultaneously in a quantum mechanical system. It focuses on the proof and application of the Heisenberg inequality by TTF. Due to the different ways and methods of practical application, the practical significance and practical value of the application are for readers' reference. Besides, the space is limited and my knowledge is limited. Finally, the development direction of FFT in quantum physics is summarized.

Keywords: fast Fourier transform, quantum field, heisenberg's inequality, uncertainty principle.

1. Introduction

The development of the fast DFT algorithm stems from the unpublished work of Carl Friedrich Gauss in 1805, when he used the DFT method to interpolate from sample observations in order to obtain the orbits of the Pallas and Juno asteroids. His method is very similar to that published in 1965 by James Cooley and John Tukey, who is often credited with inventing the modern universal FFT algorithm. Although Gauss's work predates Joseph Fourier's result, it is not named after him because he did not analyze computational timeBetween 1805 and 1965, many people improved some version of the FFT algorithm [1-4]. In 1932, Frank Yates published his interactive algorithm, an innovative algorithm for efficient computation of the Hadamard and Walsh transforms. Yates algorithm was still used for statistical design and experimental analysisIn 1942, G. C. Danielson and Cornelius Lanczos published DFT for X-ray crystallography, but the computation of the Fourier transform remained a huge bottleneck in the field. While many past approaches have focused on reducing the constant factor of O(N2) computation by exploiting "symmetries", Danielson and Lanczos realized that "periodicity" could be used and applied the "doubling trick" to "doubling with slightly more than twice the labor", Although they did not analyze that it leads to O(N log N) scaling, which is the same restriction as the Gaussian model.James Cooley and John Tukey respectively discovered and improved upon previous work [5-7], obtaining more general FFTS in 1965, since N is composite and not necessarily a power of two, and analyzing O(N log N) scaling. However, the idea was proposed by Tukey at a scientific committee headed by President Kennedy that discussed detecting Soviet nuclear tests by placing sensors around the Soviet Union. Given the need to analyze the output of these sensors, an algorithm is needed here, for which FFT fits exactly. After discussions with Tukey, Garwin realized that the algorithm was not only applicable to national security issues, but could be applied to a wider range of problems, including his current interest in determining the periodicity of the spin orientation of 3D helium-3 crystals. Gavin brought Tookey's idea to Cooley (both worked at IBM). Cooley and Tookey published the paper in a relatively short period of six months. Unfortunately, Tukey did not work at IBM, and the idea was challenged by patent applications, but the algorithm was put into the public domain, and through updates and improvements over the next decade, the FFT algorithm became a cornerstone of computational science.

The first is to illustrate how this principle applies to better understood physical environments, as it is indistinguishable at the macroscopic scale of human experience. Quantum physics uses two frameworks (wave mechanics picture and matrix mechanics picture) to explain uncertainty reasonably. Among them, the advantage of the wave mechanics image is that it is more intuitive visually, while the matrix mechanics image is more abstract, but it is more understandable to explain.

For wave mechanics, using mathematical analysis, there is an uncertain relationship between position and momentum. This is because the expression of the wave function on the corresponding two orthogonal bases in the Hilbert space is special, and they are the Fourier transform of each other. For accurate localization, the nonzero function and its Fourier transform are obviously not enough to get the result. The way to consider all Fourier analysis, however, is that there is a trade-off, namely between the Fourier conjugate variances. In quantum mechanics, by contrast, the key is that the position of the particle appears as a matter wave and the momentum is its Fourier conjugate, guaranteed by the de Broglie relation p = H k, where k is the wave number.

In matrix mechanics, any pair of noncommutative self-adjoint operators representing observations is subject to a similar uncertainty constraint. The eigenstates of the observed values represent the state of the wave function under the measured values (the eigenvalues). Let us say: if an observable a is measured, then the system is in the characteristic state Ψ of that observable. However, an observable object of A certain intrinsic state is not necessarily another eigen state of observable object B: suppose it is, then it is not the only correlation measure, because the system is not in the eigen state of the observable object.

In June 1925, Heisenberg formulated matrix mechanics. Since then, the old quantum theory has gradually declined, and the era of modern quantum mechanics has officially opened. Matrix mechanics makes the bold assumption that classical concepts of motion do not apply at the quantum level, that electrons bound to atoms do not have well-defined orbits, but rather move in obscure, unobservable orbits, and that the Fourier transform of time involves only discrete frequencies of electromagnetic radiation that can be observed due to quantum transitions. After reading Heisenberg's paper, Born discovered that Heisenberg's mathematical operation was the matrix calculus he had learned as a student, and that there was a special relationship between two infinite matrices representing position and momentum, namely regular commutation, which was expressed by the equation $[x, p] = xp - px = i\hbar$. However, they do not understand the significance of this important result, and they cannot give a reasonable interpretation. In 1926, Heisenberg was hired as a lecturer at the Niels Bohr Institute at the University of Copenhagen to assist Niels Bohr in his research. The uncertainty principle can directly interpret the regular commutation relation between position and momentum: if the measurement position does not disturb the momentum and the measurement momentum does not disturb the position, then the order relationship between the measurement position and momentum need not be concerned, and the regular commutation relation between position and momentum becomes: $[x, p] = xp - px = 0$. In 1929, Howard Robertson derived the uncertainty relation based on commutation [8].

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2. Basic descriptions

Fast Fourier Transform (FFT for short) is not a new transform, but a fast algorithm of Discrete Fourier Transform (DFT for short). For a long time, the DFT was not really used because of the large amount of computation, even if it was used by computers, it was difficult to deal with problems in real time. It was not until 1965, when J. W. Cooley and J. W. Key first proposed a fast algorithm for DFT operations, and later, when fast algorithms by G. Sullivan and J. W. Key appeared in succession, that fundamental changes took place. People began to realize some inherent laws of DFT operation, and thus quickly developed and improved a set of high-speed and effective operation methods, which is now commonly known as the fast Fourier transform FFT algorithm [1]. FFT is the fast algorithm of discrete Fourier transform, it is based on discrete Fourier transform odd, even and other characteristics, although improved discrete Fourier transform algorithm, but in theory did not break through the original Fourier transform theory, the only helpful is that discrete Fourier transform can be used in computer systems [3]. FFT generates discrete Fourier transform (DFT). Let x(n) be a finite-length sequence with length N, then define the n-point discrete Fourier transform of x(n) as [3]:

$$x(k) = DFT[X(n)] = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}\; k = 0,1,\ldots,N-1 \tag{1}$$

The inverse discrete Fourier transform (IDFT) is [3]

$$x(n) = IDFT[X(k)] = \frac{1}{N}\sum_{k=0}^{N-1} X(k) W_N^{-kx} \tag{2}$$

The wave discussed in quantum mechanics is a probabilistic wave. According to de Broglie relation, the momentum p=hλ and energy B=hv of a particle corresponding to a monochromatic plane wave y(P)(wavelength λ, frequency v). If φ(p) is used to represent the momentum distribution function of particles, w(F) and φ(β) can be obtained by Fourier expansion [7]:

$$\psi(r) = \frac{1}{(2\pi\hbar)^{\frac{3}{2}}} \iiint \phi(p) e^{ip\cdot\frac{r}{\hbar}}\, d^3p \tag{3}$$

$$\phi(p) = \frac{1}{(2\pi\hbar)^{\frac{3}{2}}} \iiint \psi(r) e^{-ip\cdot\frac{r}{\hbar}}\, d^3r \tag{4}$$

It can be seen from Eq. (3) that for a definite position F, the value of motion is uncertain, and its distribution range is determined by the integral formula. Similarly, in Eq. (4), for the definite momentum value β, the value of position strictness is uncertain. The concept of uncertainty between coordinates and momentum is introduced qualitatively through Fourier transform relation between coordinates and momentum [5].

3. Quantum mechanical illustration of Heisenberg's uncertainty principle

The Heisenberg uncertainty Principle applies to the quantum behavior of all microscopic particles. The uncertainty principle is verified by the coordinate representation and momentum representation wave function of particles, which is very helpful for us to understand the wave-particle duality and uncertainty principle of microscopic particles. For example, for the one-dimensional harmonic oscillator, its ground state wave function is

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\frac{m\omega}{2\hbar}x^2} \tag{5}$$

The uncertainty principle can be verified by analytical method, but the calculation is more complicated. One mainly uses the lift operator for verification. The definition and recursive formula of one - dimensional harmonic oscillator rise and fall operators:

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$$\begin{cases} x = \sqrt{\dfrac{\hbar}{2m\omega}}(a_+ + a_-) \\ p = i\sqrt{\dfrac{\hbar m\omega}{2}}(a_+ - a_-) \end{cases} \quad (6)$$

and

$$a_+|n \geq \sqrt{n+1}|n+1\rangle, a_-|n \geq \sqrt{n}|n-1\rangle \quad (7)$$

$$\langle x\rangle = \langle n|x|n\rangle = \sqrt{\frac{\hbar}{2m\omega}}\left(\sqrt{n+1}\langle n|n+1\rangle + \sqrt{n}\langle n|n-1\rangle\right) \quad (8)$$

By orthogonal normalization condition $\langle n|n'\rangle = \delta_{nn'}$, one has to

$$\langle x\rangle = \sqrt{\frac{\hbar}{2m\omega}}\langle 0+0\rangle = 0 \quad (9)$$

By the same token

$$\langle p\rangle = i\sqrt{\frac{\hbar m\omega}{2}}\langle n|a_+ - a_-|n\rangle = 0 \quad (10)$$

For the square mean of coordinates

$$\langle x^2\rangle = \langle n|x^2|n\rangle = \frac{\hbar}{2m\omega}[0+n+n+1] = \frac{\hbar}{m\omega}\left[n+\frac{1}{2}\right] \quad (11)$$

Similarly to the momentum of the mean,

$$\langle p^2\rangle = \langle n|p^2|n\rangle = -\frac{\hbar m\omega}{2}[0-n-n-1] = \hbar m\omega\left[n+\frac{1}{2}\right] \quad (12)$$

The above results into the variance of coordinate and momentum formula:

$$\begin{cases} (\Delta x)^2 = \langle x^2\rangle - \langle x\rangle^2) \\ (\Delta p)^2 = \langle p^2\rangle - \langle p\rangle^2 \end{cases} \quad (13)$$

Hence, $\Delta x\Delta p = \hbar(n+\frac{1}{2})$. Since the quantum number n takes the value 0,1,2,... , so

$$\Delta x\Delta p \geq \frac{\hbar}{2} \quad (14)$$

This result is in conformity with completely.

4. Applications

4.1. The usage of Heisenberg's uncertainty (The use of quantum computers)

As is known to all, a classical computation can be described as the physical evolution of an input signal sequence through a deterministic algorithm (logic gate operation), based on the deterministic nature of standard bits that are not 0 or 1. Classical algorithms are implemented by adding logic to the internal logic of a classical computer, or a classical Turing machine. On the other hand, the quantum super computation is based on a coherent superposition of qubits 10> and 1>. According to the requirements of quantum algorithm, the input signal can be described by quantum superposition state and unitary transformation, that is, quantum logic gate operation. This is a human controlled process that takes the input state as the initial state of the quantum measurement and the final state, the output state, giving the result of the quantum computation. The concept of a quantum computer originated in the 1960s and 1970s, when reversible computers were developed to overcome the problem of energy consumption. The combination of the heat of computer motion and the shadow ring of computer motion greatly limits the speed of computer operation. To put it simply, when the circuit integration density is very high, that is, when △x is small, △p will be large, and the quantum interference effect described by quantum

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physics will occur, which will destroy the function of traditional computer chips. Only unitary transformations in quantum mechanics can be truly reversed. The study of quantum computers became physics in 1995, when Scholl proposed a large number of factor quantum algorithms and proved the possibility of quantum computing in cooling ion systems [9].

4.2. Quantum algorithm based on Fourier Transform

The problem of solving the periodicity of a function ($f(x) = a^x \, mod \, N$) can be done in polynomial steps using QFT. Shor's algorithm uses this principle to transform the famous NP problem of prime factorization of large numbers into P problem. At present, the time complexity of the effective traditional factorization algorithm is $O\left(exp\left[n^{\frac{1}{3}}(lg\, n)^{\frac{2}{3}}\right]\right)$. On the quantum computer, the time complexity of finding the two prime factors of n-bit large numbers using Shor's quantum algorithm is$O\left(n^2(lg\, n)(lglg\, n)\right)$, and is a polynomial of n. It can be seen that an exponential speedup over the classical algorithm is achieved using the Shur quantum algorithm. The quantum Fourie transform can be used to solve not only the period of a periodic function, but also the hidden subgroup problem, and the problems of order, period, discrete logarithm and other problems can be reduced to the special cases of the hidden subgroup problem. Jozsa first gave a unified description of D J algorithm and Shor algorithm in the form of implicit subgroup [10-12].

4.3. Quantum algorithm

The two most successful quantum algorithms are quantum Fourier transform algorithm based on Shor and quantum subsearch algorithm based on Grover. The first kind of algorithm includes large number factorization algorithm and discrete logarithm algorithm [13]. This class of algorithms provides periodic problems which are actuallyfinding periodic functions and can be reduced to implicit subgroup problems. The quantum search algorithm based on Grover provides the root square acceleration of the classical algorithm and has been developed into a quantum search algorithm system. The importance of quantum search algorithm lies in the wide application of search technology in classical algorithms. Many of these applications can be solved with accelerated quantum algorithms that are faster than classical algorithms. Since the Shor factorization algorithm and Grover search algorithm were proposed, many researchers have done a lot of research in the field of quantum subalgorithm, but no significant breakthrough has been made so far [14].

4.4. The Heisenberg uncertainty is used for quantum entanglement

As it known to all, a real quantum system is inevitably affected by the surrounding environment, which directly leads to the decoherence phenomenon in the entangled state of the quantum system, thus bringing uncertainty to the application of quantum information. Therefore, it is very valuable to explore the interference mechanism of entanglement of quantum system caused by some random factors due to the influence of external environment. Recently, using Milburn's intrinsic decoherence theory [15], the entanglement properties of Heisenberg spin chains in the presence of intrinsic decoherence have been studied intensively. Li et al. discussed the influence of external magnetic field on entanglement properties in the presence of intrinsic decoherence in a three-particle Heisenberg chain. Qian studied considering intrinsic decoherence, such as Dzyaloshinskii Moriya Ising model (DM) interaction on the two particles entanglement degree of negative impact; Xu et al. studied the influence of inhomogeneous applied magnetic field on the entanglement of two anisotropic Heisenberg XYZ chain when intrinsic decoherence is considered. Xie et al. discussed the influence of anisotropy on entanglement properties in the presence of intrinsic decoherence of a two-particle Heisenberg XYZ chain. Zhang et al. discussed the evolution of entanglement with time in the presence of intrinsic decoherence of a two-particle Heisenberg XY chain. Guo et al. discussed the dynamic evolution law of quantum entanglement in the two-particle XXZ Heisenberg system under the influence of various external factors under the consideration of intrinsic decoherence, and concluded that intrinsic decoherence has obvious inhibition on the system's entanglement.

4.5. The development direction of FFT applied to quantum physics

Now under development is quantum computing and its latest development direction quantum intelligent computing, quantum intelligent computing is still in the early stage of development. Future research directions include following. For specific problems, traditional intelligent computing and quantum theory are combined to construct problem-oriented quantum intelligent computing methods, image segmentation algorithm using quantum intelligent computing for image processing, etc. Developing and improving the existing quantum intelligent computing system is also necessary. Although many researchers have studied in this field and achieved some results, it is still necessary to study the combination of quantum theory and traditional intelligent algorithms in a deeper way, and the research in some fields is still in the blank. Rigorous mathematical analysis of the performance of quantum intelligent computing algorithm is also needed. At present, some quantum intelligent computing algorithms have better performance than traditional intelligent computing algorithms, but the convergence of the algorithm and the computational complexity need further analysis and research

5. Conclusion

In summary, this paper studies the application of FFT in quantum mechanics, focusing on the use of fast Fourier transform to prove Heisenberg's uncertainty principle and the development of TTF. A method for proving Heisenberg's uncertainty principle using TTF is proposed. Since many commonly used proof methods are referenced, the proof process is somewhat similar to the commonly used methods, but the proof method in this paper still has certain research significance. Then the application of Heisenberg's uncertainty principle is discussed. However, there are still certain limitations to the content studied in this paper. Firstly, quantum physics and fast Fourier transform (FFT) are two completely different fields. FFT is a mathematical algorithm used to convert a signal or function from time domain to frequency domain. It has a wide range of applications in signal processing, image processing, communication and other fields. However, FFT itself is not a tool for studying the limitations of quantum physics. Quantum physics is a branch of physics that studies the behavior of the micro world, mainly focusing on micro fields such as atoms, molecules and elementary particles. The topics involved include quantum mechanics, quantum field theory, quantum information and so on. The limitations of quantum physics are mainly related to our understanding and description of the micro world, rather than with mathematical algorithms such as FFT. This paper only uses FFT to prove Heisenberg's uncertainty principle in numerical solution methods. Therefore, future research can explore the use of FFT in quantum physics from the following aspects: 1. Quantum computing and quantum information: Quantum computing is a new computing model that uses quantum bits (qubits) in quantum mechanics for computation. It has the potential to be more efficient than classical computing in some problems. In the research of quantum computing, some numerical methods and algorithms may use FFT or similar techniques to process the evolution of quantum states and quantum gate operations.

2. Quantum communication and quantum cryptography: Quantum communication and quantum cryptography are research fields that use quantum entanglement and quantum properties to achieve secure communication. The transmission and measurement of quantum states are key issues in these fields, and some technologies may use FFT or other numerical algorithms to process the performance and efficiency of quantum communication systems.

3. Quantum simulation and optimization: Quantum simulation uses a quantum system to simulate the behavior of other quantum systems, which helps to study complex quantum systems and material properties. Quantum optimization uses quantum computing to solve optimization problems, some of which may use numerical techniques such as Fourier transform.

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