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Research on the uncertainties of black hole spin measurement

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Abstract. Black hole is an important object and research object in astronomy and spin is one of the three physical quantities used to describe them. It's vital to get the spin parameter of a black hole as accurate as possible. Recently, researchers have used many methods to measure the spin parameter and two of them are most commonly used, the continuum-fitting method and the reflection-fitting method. It's difficult to avoid errors in the measurement of the two methods, but we can use various methods to minimize the error in the measurement. In this paper, we try to figure out using the two methods in measuring what kind of black hole's spin can we reduce errors. We consider both the systematic error caused by model assumptions and the accidental error caused by dynamical parameter. Finally, we conclude that both the two methods can only measure black holes with certain accretion rate between 0.01 - 0.3. The continuum-fitting method can reduce errors in measuring high-spin black hole with small viscosity of the accretion disk. The reflection-fitting method can reduce errors in measuring high-spin/ low corona-height black hole.

Keywords: Black Holes, Accretion Disk, X-Rays.

1. Introduction

As we all know, black hole is one of the most important astrophysical objects and it can be fully characterized by three physical quantities: electrical charge, mass and angular momentum according to no-hair theorem [1]. In most of the cases, we assume charge neutrality and the space-time can *be* described by Kerr metric. Using these quantities, we can describe the spin of black hole. The spin of black hole is related to a lot of astrophysical phenomena, such as the relativistic jets that produced by black hole. Usually, we use a dimensionless parameter a_* to represent the spin and a_* is defined by

$$a_* \equiv a(M)^{-1} = cJ(GM^2)^{-1} \quad (1)$$

where M and J represent the mass of a black hole and the angular momentum of a black hole. G is Newtonian constant of gravitation and c is the speed of light.

To measure spin parameter of black hole, we now mainly use two methods: the continuum-fitting method and the reflection-fitting method. Both the two methods use R_{isco} to calculate the spin parameter based on the relationship between R_{isco} and a_*.

R_{isco} is the innermost stable circular orbit. In black hole space-time, circular orbits are only stable when its radius greater than R_{isco}. Mostly, the radius of the accretion disk is large enough so that we can use Newtonian mechanics to approximate the calculation. The closer the orbit is to the black hole, the faster the material in the orbit moves. However, the spin of black hole causes a frame-dragging

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effect. As a result, friction occurs between adjacent orbits, letting the outer orbit lose energy and move closer to the black hole. When the radius of the orbit reaches Risco, it can't remain stable anymore. Any small perturbation sends matter rapidly spiraling closer to the black hole. This shows that the spin of a black hole influence R_{isco}. And the relationship between spin parameter and R_{isco} can be given by

$$R_{isco} = \{3 + Z_2 \mp [(3 - Z_1)(3 + Z_1 + 2Z_2)]^{1/2}\} r_g \tag{2}$$

where we assume orbits restricted to the $\theta = \pi/2$ plane and the sign $\mp$ is due to prograde retrograde orbits [2]. Then we define,

$$Z_1 = 1 + (1 - a_*{}^2)^{1/3}[(1 + a_*)^{1/3} + (1 - a_*)^{1/3}] \tag{3}$$

$$Z_2 = (3a_*{}^2 + Z_1{}^2)^{1/2} \tag{4}$$

Using this formula, we can calculate spin parameter of certain black hole by calculating R_{isco}. Recently researches have successfully used the two methods to get R_{isco} from observation data and then calculate spin parameter, but we can hardly avoid uncertainties and errors. Thus in this paper, we mainly study the uncertainty of the two methods of the spin parameter.

The paper is organized as follows. In section 2 we explain the theory of continuum-fitting method and reflection-fitting method. Section 3 mainly discusses some of the uncertainties in the two methods and explores in measuring the spin of which kind of black hole can we reduce uncertainties as many as possible. In section 4 we conclude the two methods can have less error in measuring some black holes. Section 5 briefly summaries the paper and presents our prospects for further research.

2. The principle of the two methods

Before we talk about the theory of the two methods, we need to know the component of the spectra of a black hole. Nowadays, we can learn some characteristics about black holes by observing the X-ray radiation they emit. The X-ray spectra we observe often contain a variety of components, and they come from different sources.

First, the friction between the gas layers in the accretion generates heat. Just a few hundred miles from the black hole, friction heats the gas up to 10 million degrees. At this temperatures, any material would give off X-rays which is observed as thermal component. And at present, the lamp-post model is generally accepted [3]. It assumes that there are some high-energy electrons near the black hole, which inverse compton scattering the photons from the accretion disk. And the high-energy electrons are called "corona".

The X-ray spectra from corona can be described as power-law and cover across the entire band from 1keV all the way up to at least 100keV. Some of the radiation emitted from the corona will react easily with a large amount of material of the disk. As a result, we can end up seeing a large amount of the interacting components called reflection component. Among those reflection component, the 6.4keV iron emission line is prominent. And the fluorescent iron photons are produced during the process that X-ray shines on the matters in the accretion disk. This process causes electrons in the K-shell of the iron ions to ionize, and electrons in the L-shell later transition back into the K-shell.

Having known the component of the spectra, now we will talk about the continuum-fitting method. As we have mentioned above, the accretion disk itself will emit X-ray and let the spectra has a thermal component. Thus, we can measure R_{isco} of the black hole by fitting the thermal spectra emitted by the accretion disk.

First, we can easily get flux of the black hole F from observation. Then using the data of the source distance of black hole D and the accretion disk inclination of black hole i, we can obtain the luminosity of the black hole L according to $L_{cosi}(4\pi D^2)^{-1} = F$ [4].

And the luminosity is proportionate to the R_{in}^2 and T^4, where T is the peak temperature of the emitted radiation and R_{in} is the inner radius. In continuum-fitting method we assume that $R_{isco} = R_{in}$. As for T, we can get it by building the thermal spectra model, because when a_* increases, the temperature of the emitted radiation will increase. (we will discuss it in section 3).

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And because the unit of R_{isco} is r_g, we also need the black hole's mass M to get r_g. In a word, using M, i, D and modeling the thermal spectra, we can get the spin parameter a_*.

As for the reflection-fitting method, the key point is the iron line. We have known that the 6.4keV iron emission line comes from the accretion disk. Because of the orbital motion of the matter in the accretion disk, the iron line is influenced by Doppler effect. Meanwhile, due to the spin of the black hole, relativistic beaming and gravitational red-shifting also have an effect on the iron line [5]. As a result, the iron line is broad, skewed line profile. As the spin changes, all of these effects change and the iron line will show different forms and we can obtain R_{isco} as well as a_* by modeling the iron line spectra.

3. Discussing the errors

Through the previous discussion, we know that the two methods model different radiation component to measure a_*. Both of them have certain assumptions in the process of fitting, and we now try to reduce the error of the two methods.

One of these criteria is the Eddington accretion rate $\dot{M}_{edd}$. And we define $\dot{M}_{edd}$,

$$\dot{M}_{Edd} = L_{Edd}(\eta c^2)^{-1} \tag{5}$$

where η is the radiative efficiency of the accretion flow and L_{edd} is the standard Eddington luminosity where radiation force fully balances gravity for fully ionized hydrogen. And usually we use $\dot{M}$ to define accretion rate.

For accretion rate in range of $\dot{M}$ ~ 0.01 - 0.3 $\dot{M}_{edd}$, the accretion disk can be considered as a geometrically-thin and optically-thick accretion disk. Only in this condition, we can assume that R_{isco} = R_{in} without making too much errors and we can use continuum-fitting method to get R_{isco}. As for the reflection-fitting method, it assumes that the iron line emission suddenly ceases at R_{isco} and the gas inside R_{isco} will not emit radiation. This also require a geometrically-thin and optically-thick accretion disk.

The errors of continuum-fitting method mainly come from two aspect, errors due to selected parameters such as M, D, i and errors in model used to fit the thermal spectra.

Usually, we used a model created by Novikov and Thorne to model the disk emission (also called the NT model) [6]. And this standard model assumes a geometrically-thin and optically-thick accretion disk as well as viscous torque vanishes at R_{isco}. We have talked about the former above. In reality, the stress responsible for angular momentum transport in a thin accretion disk is likely to be magnetic which means that viscosity α always causes errors. And according to the previous research, the error of spin parameter a_* is larger when the α is larger [7]. Also, the error becomes smaller as the spin parameter a_* is larger.

In real life measurements, we use KERRBB2 model to fit the thermal component. KERRBB2 model is the combination of two models, BHSPEC and KERRBB [8-10]. KERRBB model is a straightforward implementation of the analytic NT model. However, the radiation emitted by the accretion disk can also react with the matters on the accretion disk and create errors, which can be reduced by BHSPEC model.

Besides the thermal component, we still need to model three components. A reflected component, a Compton component and a low-energy cutoff. And we know that black hole has three outburst states, thermal, hard and SPL. In thermal state, the accretion disk dominates the spectra. As a result, the reflected component as well as the Compton component are relatively weak enough to be ignored.

However, compared with the errors due to selected parameters M, D, i, the errors in model are much smaller and the difference between the two errors is usually an order of magnitude. Therefore, the errors are dominated by parameters. This part of errors can be computed in detail via Monte Carlo simulations.

Compared with continuum-fitting method, reflection-fitting method does not need extra parameters M, D, i, which can reduce part of the errors. Because the reflection spectra depend on the lamp-post model of the black hole and the iron line is twisted by Doppler effect, relativistic beaming, gravitational red-shifting. As a result, the spin measurement uncertainty is naturally influenced by the lamp-post model. Moreover, the X-ray spectra of a black hole is complex and the spectra we have observed is always affected during its propagation process in the universe. And this part of influence mainly comes from the galactic medium. They absorb some of the radiation. Last but not least, if there is a strong disk wind in a black hole binary system, some of the radiation will also be absorbed and in return we cannot observe the true reflection spectra on earth.

The lamp-post model assumes some high-energy electrons called "corona" above the black hole and accretion disk. The closer the corona to the black hole, the stronger the relativistic distortions are. According to the previous research, The closer the corona to the black hole results in an accurate spin measurement [11]. When a_* is larger, the spin of black hole increases, which can also result in stronger relativistic distortions. As a result, for a certain high-spin/ low corona-height black hole ($a_* > 0.8$, corona height $h < 5\ r_g$), we can obtain a more accurate measurement of the spin parameter a_* [12]. Besides, the absorption caused by the galactic medium often affects the low energy segment of X-ray spectra and it causes less influences on the reflection component. It won't cause the reflection-fitting method to make a lot of errors. And for the disk wind, this part of absorption will leave a "pit" in the X-ray spectra. we can easily find it in the spectra and reduce errors by fine-tuning the reflection fitting model.

4. Results

Although the reasons for the errors in the two methods are different, but we are able to reduce the error of both the two methods when measuring certain black hole's spin.

For the continuum-fitting method, we can reduce errors when we use it to measure a high-spin black hole's spin parameter. Also, smaller viscosity α are helpful in reducing errors according to what we have talked in section 3. And certain accretion rate of a black hole accretion disk makes sure a geometrically-thin and optically-thick disk which is the key assumption of the continuum-fitting method. Moreover, the continuum-fitting method needs to wait for the black hole to reach a thermal outburst state. In thermal state the radiation emitted by corona is small enough to be ignored, so the reflected components which is caused by the accretion disk reflects the radiation from corona can be ignored, too. But according to section 3, the most important part of errors of continuum-fitting method is the selected parameters M, D and i. And the black hole's mass M is vital for measurement of spin parameter. Therefore, continuum-fitting method can only be used in measuring a thermal outburst stellar mass black hole. On one hand, the mass of stellar mass black hole can be measured relatively accurate by existing observation and measurement techniques. While we can hardly determine the accurate M of supermassive black hole and black hole candidate. On the other hand, the more massive the black hole, the cooler the accretion disk. The radiation emitted by these accretion disk is often in the optical and ultraviolet bands. These bands face severe absorption problem. Therefore, we can hardly use continuum-fitting method to measure its spin. Compared with the continuum-fitting method, the reflection-fitting method has a wider scope of application. Its errors won't be bound by selected parameters M, D, i. And we can also reduce its errors by measuring a high-spin and low corona-height black hole. Besides, measuring black hole with accretion rate between 0.01 - 0.3 M_{edd} can also reduce errors. Neither of the two methods can fully eliminate errors, some statistic errors cannot prevent and what we can do is just create as accurate a model as possible.

5. Conclusions

We have talked about the theory of continuum-fitting method and reflection-fitting method as well as the errors of them. At the same time, we know that both the two methods have its limits. One of the limits is the standard accretion disk model. It makes the accretion rate a prerequisite for the ability to measure spin using either method. Also, it's vital to improve existing standard model or create new model that fit better. Besides, new methods have been used to measure the spin of black hole such as, measuring spin from quasi-periodic oscillations, gravitational waves and so on. Eventually we may have the ability to measure the spin of every black hole with different method.

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