Fractal theory analysis of lung CT image recognition technology and its parameters

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Abstract. With the popularity and maturity of medical imaging technology, CT scans, and other medical technologies are increasingly utilized in clinical diagnosis and treatment. In the diagnosis and treatment of pneumonia or lung tumors, clinicians make extensive use of this technology and rely on CT images to assess the progression of disease or the effectiveness of treatment. But the accuracy of visual interpretation is limited, and much of it requires a doctor's extensive clinical experience. In order to reduce the workload of doctors and the misdiagnosis rate, fractal theory is considered an important tool in medical image analysis. This paper introduces the fundamental definition of fractal theory, its objectives, and how the two fractal parameters fractal dimension and lacunarity function to facilitate the recognition, and some of their advantages. They strongly reflect the actual tumor development, and highlight the crucial role played by fractal theory in medical treatment.

Keywords: Fractal Theory, Lung CT Image, Lung Tumors.

1. Introduction

Medical imaging, which may be used to visualize the interior of bodies, is a method frequently utilized in the field of diagnosis and therapy today. The importance of such a technique can be understood through an example of taking a CT computed tomography) scan of the brain for a patient with a tumor inside it. Surgery could even occur remotely with medical imaging after the tumor's location is pointed out. The operation is to attach a source that emits waves and detectors onto a human body. Different kinds of waves are used in this case, and the degree of absorption of waves varies inside the body. Finally, the image is developed by composing all degrees of wave absorption by tissues to see whether there is a problem related to those tissues [1, 2]. Medical imaging is becoming well-developed due to its high detection sensitivity, non-intrusiveness to human bodies, and relative simplicity of machine operation. During the COVID-19 pandemic, nucleic acid testing is the preferred method to detect whether people get infected. However, it was later regarded that taking a CT test is way more efficient at seeing the exact condition of the lungs. If scientists have noticed patterns for people being infected, then it would be, in turn, simple to diagnose [3]. However, there exists a problem of low resolution when observing the images with the human naked eyes, and it is highly dependent on the clinical experience of doctors. To reduce the possibilities of misdiagnosing due to the unclear image recognized by clinicians, they require a more sophisticated and impersonal method to utilize all information from a single image.

As a result, fractal geometry is functioning in medical imaging. Fractals are imported because they are a valuable tool when dealing with irregular patterns and uneven metric spaces. This happens inside human bodies, where a dynamical system exhibits chaotic behavior. The organs and tissues do not follow any regular or expected patterns; they are considered non-linear and multidimensional. However, under examination, they are showing properties of self-similar and scale-free [4]. Self-similarity means it is similar between a part and the whole, while the organs are scale-free, revealing that if some scale enlarges a region, it can map onto the original shape. With the aid of fractal geometry, clinicians can resolve the medical images with complexity shown on the branches of the organs, which helps gain a better understanding of the chaos in body systems. Meanwhile, the textural features can also be characterized by fractal dimensions. It is interesting to find out that although there may be more than one figure associated with the dimensional quantities, they might finally coincide due to various definitions for dimensions, which leads to a more coherent result.

The research aims to discover how fractals are applied to different medical images by clearly defining fractal geometry and some good examples of fractal objects, showing that the lungs are perfect candidates for fractal analysis. It is also crucial to introduce two critical fractal parameters, which characterize the degrees of roughness and self-similarity from the surface of objects and the spatial distributions. Consequently, there will be real-life examples related to how these parameters are responsible for collecting information for patterns of lung tumors and, therefore, providing accurate justification for medical treatment.

2. Concept of fractal geometry and fractal dimension

Generally, fractal geometry is used for quantifying structures that Euclidean geometry can hardly represent. The structures are sometimes intricate inside either mathematical or natural fractal objects, where property such as self-similarity is still shown. Although Euclidean geometry is more routinely used in medical imaging, it is relatively simple, and complexity is only amenable to measurement and treatment with fractal geometry. Moreover, fractal geometry can also be used to classify biological structures based on the parameters widely defined in fractals.

There are three classic characterizations of fractals [2]. The first one is self-similarity, where any more minor part inside can be a replication of the whole. Take the Koch curve as an example. There are miniature replicas of sizes $1/3$, $1/9$, and $1/3^n$ of the whole, and those parts are said to be self-similar to the whole pattern. From this example, the second property of fractals can also be revealed, which is about scaling. That means if those smaller parts are magnified to corresponding scales, they will be invariant to the original. Lastly, fractals have their fractal dimension, which could be non-integer and differ from the definition of Euclidean dimension.

As fractal dimension can be non-integer, there is also a name called fractional spatial dimension. It relates two quantities: the metric measured and the scale used. In most cases, the smaller the scale, the more accurately the metric is measured. This is because tinier units suggest more complexity of an irregular curved shape, and this concludes that such fractal objects do not have a fixed length.

Because fractal dimension is a quantitative parameter, it can provide information on the properties of self-similar objects. Another usage for fractal dimension is that it can quantify either short-range or long-range complexity for temporal series, which is suitable for regular or irregular time intervals. The mechanism is the following. Doodle a curve connecting each value under the given measure across time. The short-term complexity rises as the number of details increases, and the data occupies more planes when the time series starts to vary [5]. This process describes how fractal dimension increases. The other emphasis is that when the fractal dimension becomes more remarkable, it suggests more short-range details. Therefore, a relatively lower fractal dimension indicates the presence of long-range variations.

3. Define fractal objects and instances

Fractal geometry is essential to deal with structures containing recursive details. To be precise, a fractal object, or a fractal set is one that is irregular enough to avoid being classified using Euclidean geometry, and again has the property of self-similarity. There are some examples of mathematical and natural fractal objects. For the former, the Koch curve and Sierpinski triangle are two classic patterns, and they are perfect as they are infinitely self-similar. In the real world, there exist wonderful patterns that belong

DOI: 10.54254/2753-8818/14/20240918

to natural fractal objects, but they have to fulfill the following, exhibiting fractal properties to the limited range of scales, and being fractal rather than in a strict geometrical sense. Objects such as clouds, and the coastlines meet the criteria, as well as the lungs as biological objects. Actually, the lung is one of the organs that is especially suited to fractal analysis, because the structure under CT scans follows the properties mentioned, and is too complex to be defined by classic geometry.

4. Calculation of fractal dimension

There are various definitions for fractal dimension, box-counting dimension, correlation dimension, similarity dimension, Hausdorff dimension, and so on. The more chosen one would be the use of box-counting dimension, in the medical field, it is easier to calculate and apply to the CT image of the lung [5]. To define box-counting dimension, it is assumed under a non-empty subset P, Nr is the number of boxes needed to cover the entire shape, with side length r to cover P, where $r > 0$. If there exists a parameter D when the side length of boxes r approaches 0, where the equation

$$\mathrm{N_r} = \mathrm{c} \cdot (\frac{1}{\mathrm{r}})^{\mathrm{D}}, \tag{1}$$

Here, with c being a constant, then D is supposed to be the box-counting dimension of P.
Usually, c takes a value of 1, and by taking logarithms on both sides and rearranging:

$$\mathrm{D} = \lim_{\mathrm{r}\to 0} \frac{\log \mathrm{Nr} - \log \mathrm{c}}{\log \frac{1}{\mathrm{r}}} = \lim_{\mathrm{r}\to 0} \frac{\log \mathrm{Nr}}{\log \frac{1}{\mathrm{r}}}. \tag{2}$$

There is how D is calculated. A preferred way of obtaining this D is to plot $\log Nr$ against $\log \frac{1}{r}$, where D is the slope of the resulting line.

However, the following called the differential box-counting (DBC) method is especially for finding the fractal dimension for gray-scale images which are three-dimensional [6, 7]. Suppose there is a square image P with the 3D coordinate system (x, y, z), the image plane can be denoted by (x, y), and the size is M×M, while z denotes the gray level intensity. Then the 2D position of the square P is partitioned into some non-overlapping grids of size $s \times s$, where s should be integers and satisfies $2 \leq \mathrm{s} \leq \mathrm{M}/2$. Onto each grid, there is a column of boxes with volume being $s \times s \times h$, where h is the height of the box. If G represents the total number of gray levels, there exists a relation $\left\lfloor \frac{G}{h} \right\rfloor = \left\lfloor \frac{M}{s} \right\rfloor$. Before coming to the fractal dimension, two more equations are needed. By assuming l and k to be the maximum and minimum number of the gray levels of the image P in an (i, j) grid, the number of boxes covering the grid is

$$\mathrm{n(i,j)} = \mathrm{l} - \mathrm{k} + 1, \tag{3}$$

and in turn the number of boxes covering the whole image P is

$$\mathrm{N} = \textstyle\sum_{\mathrm{i,j}} \mathrm{n(i,j)}. \tag{4}$$

Finally,

$$\mathrm{FD} = \text{Fractal Dimension} \approx \frac{\log N}{\log \frac{1}{s}} \tag{5}$$

To be precise, it is better to plot $\log N$ against $\log \frac{1}{s}$, and use the least-square fitting line to calculate the slope.

However, there is also an improved way to compute N, the number of boxes to cover the whole image, where the step to calculate FD remains the same [8, 9]. It is suggested that using the variance of the gray level can compensate more for the not even distribution of the gray level gradient. Therefore, it is more appropriate to use the square root of the variance, which is also the standard deviation of the gray levels in a grid, than the difference between the maximum and minimum gray levels falling onto the grid. The variance is defined as

$$\delta^2 = \frac{1}{r^2} \cdot \sum_{k=1}^{r^2} (I_k - \bar{I})^2. \tag{6}$$

Then

$$\delta = \frac{1}{r} \cdot \left(\sum_{k=1}^{r^2} (I_k - \bar{I})^2\right)^{\frac{1}{2}}. \tag{7}$$

Substituting it back to the formulae of n and N can obtain number of boxes covering the grid and the entire image respectively:

$$n_r = \frac{1}{rh} \cdot \left(\sum_{k=1}^{r^2} (I_k - \bar{I})^2\right)^{\frac{1}{2}} + 1, \tag{8}$$

$$N_r = \frac{1}{rh} \cdot \sum_{i=1}^{r^2} \left(\sum_{k=1}^{r^2} (I_k - \bar{I})^2\right)^{\frac{1}{2}} + r^2 = \frac{1}{G} \cdot \sum_{i=1}^{r^2} \left(\sum_{k=1}^{r^2} (I_k - \bar{I})^2\right)^{\frac{1}{2}} + r^2. \tag{9}$$

The fractal dimension is again

$$\mathrm{FD} \approx \frac{\log N_r}{\log \frac{1}{s}}. \tag{10}$$

Here, r is a parameter imported to represent M/s, and r^2 is the total number of pixels in a box. I_k is the gray level at the kth pixel, and $\bar{I}$ is the mean of the gray levels of all pixels from the boxes.

The fractal dimension calculation methods are introduced here because they can spot the differences between malignant and benign lung tumors. The fractal dimension is similar to the human visual system concerning the ability to assess objects. For example, the greater the roughness of the object's surface identified by humans, the more significant the value of the fractal dimension, so this is why the fractal dimension is said to quantify the amount of complexity and roughness [3].

The above information also suggests that a change in the fractal dimension value can monitor the traceable development of the sizes or volumes of the pulmonary nodules. Moreover, the measurement of the fractal dimension of both inner microvessels and external contour of the tumor can be critical clinical indicators of the development of tumors as well, as it is a process of deepening into the structure and texture of tumors. As a result, uses of fractal dimensions are becoming more reliable in giving predictions and treatments for diseases [4].

On the other hand, some may realize the fact that values of fractal dimension are sometimes quite similar, or even identical, between malignant and benign nodules, and there could be many CT images that have a fractal dimension within a small range of values, which means the amount of data is overwhelming. Using fractal dimension is limited, but it is reasonable to use fractals to roughly classify the CT images at first by pointing out those with close values of fractal dimension and then comparing their multifractal spectrum to subdivide different nodules in the lung parenchymal tissue. In conclusion, this combination of techniques can provide a more precise insight into the characteristics of different nodules, and it is also highly efficient [2].

There may be more complicated applications of fractal analysis or fractal dimension in the recognition or diagnosis of cancers, for example, the decomposition of histopathological images. The tool involved here is called a clinical decision support system, where the general function is to exploit the subbands and extramural fractal characters for the best basis selection of the meningioma brain histopathological image classification. The subbands selected here have the roughest surface, which equally has the highest fractal dimension, and then discard other subbands from the same level [10]. Subsequently, this application shall be transferred to the analysis of lung cancer histopathology.

To obtain the fractal dimension value, it is only sometimes realistic to do it by hand by counting the number of boxes. As this kind of algorithm is box-size sensitive, researchers have developed programs that can generate a similar fractal dimension value, such as FAuNSs, to facilitate the recognition and even capture the cancer cells from the blood sample [11].

DOI: 10.54254/2753-8818/14/20240918

5. Another important fractal parameter--lacunarity

As introduced above, fractal dimension only manages to show the degree of complexity, it cannot describe the actual characteristics of the texture of objects. Therefore, lacunarity is imported here as a geometric measurement to display meta-information about the difference of textures under distinct scales of either fractal or nonfractal images [3]. In other words, it can illustrate the distribution or size of the gaps of objects, where the gaps are the leftovers when an object is filling the space [5]. Together with the fractal dimension, these two parameters are able to well define some main properties of an irregular object.

More importantly, the lacunarity measure can quantify the rotational or translational invariance. So a higher lacunarity implies the more rotationally variant of the shape, which means the shape appears to be more different after rotation. This in turn concludes that the shape is of higher heterogeneity, meaning the gaps are more variant, and the texture will be less uniform.

Methods of calculation can be similar to the box-counting from fractal dimension, one of the most popularly used is called the gliding box algorithm. A gliding box is a box of a specified size that travels through the image. The rule is the gliding box going pixel by pixel following a certain order until it covers the full image. There is no decomposition of the image to adjacent boxes, corresponding to the calculation of fractal dimension. Eventually, the gliding box is centered at each pixel, and there the number of holes within each pixel is counted, where these values will increment a histogram value. With this histogram showing the distribution of boxes appearing in the image, the measure of lacunarity is characterized and extracted. It is denoted that X is the number of holes inside pixels [12].

$$\text{Lacunarity} = \frac{\mathrm{E}(\mathrm{X}^2)}{(\mathrm{E}(\mathrm{X}))^2} = 1 + \frac{\mathrm{Var}(\mathrm{X})}{(\mathrm{E}(\mathrm{X}))^2} \tag{11}$$

where $E(X)$ represents the expected number of holes, and $\mathrm{Var}(\mathrm{X})$ for the variance of number of holes.

Although the fractal dimension is performing well in distinguishing either malignant tumors at different phases or different types of nodules, this parameter characterizes the geometric complexity only. Much more details of the image are hidden and therefore other indicators for radiomics are required for research and lacunarity is included [13]. From one experiment, it can be detected that at each stage, the higher the value of fractal dimension, the lower the lacunarity, and therefore the greater the homogeneity of the tumor. This observation of trends from data can also deduce that aggressive tumors may be more homogeneous [14].

6. Conclusion

Starting with the extensive application of medical imaging, this paper aims to present how fractal theory can serve as a valuable tool to facilitate the analysis of medical images such as CT scans. This paper introduces fractal geometry and its three characteristics: self-similarity, scale invariance, and fractal dimension. It gives examples of objects suitable for fractal analysis and explains why the lung structure is an ideal candidate for fractal observation. Secondly, two important fractal parameters are derived: fractal dimension and lacunarity. The assessment of image irregularity mainly relies on fractal dimension calculation, so this paper lists the commonly used dimension calculation methods along with the improved ones to enhance accuracy.

Additionally, researchers refer to other radionics indicators, such as lacunarity, to generalize image texture features. As another proposed fractal parameter, it focuses on the distribution of texture or image voids, which, together with the fractal dimension, can provide an insight into the actual lung tumor status or distinguish between benign and malignant nodules. One conclusion from experiments suggested that the higher the fractal dimension value and the lower the lacunarity, the lower the tumor heterogeneity, given a specific tumor growth stage.

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