

An overview of BRDF models in computer graphics

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Abstract. This paper aims to provide an overview of the Bidirectional Reflectance Distribution Function (BRDF) and the various parametric numerical models, known as BRDF models, used to make it easier and more efficient to use BRDF data in computer graphics. It introduces the basics of BRDF and classifies and describes some well-known BRDF models, including their advantages and disadvantages. The paper also covers the concept of data-driven models, which can achieve a high degree of realism by directly acquiring and using measured BRDF data in the rendering process.

Keywords: BRDF, empirical models, physically-based models, data-driven models.

1. Introduction

In computer graphics, creating realistic images is one of the most important areas of focus, particularly in industries such as film production and game development that aim to achieve realism. Researchers have been trying to make rendered images look as similar as possible to real-world objects.

It is essential to effectively describe the interactions between light and objects to generate realistic images. Factors such as the material of an object, its geometric shape, and others can result in different interactions between the object and light. In the field of computer graphics, the Bidirectional Reflectance Distribution Function (BRDF) is an equation used to describe how a surface reflects light. As the name suggests, the BRDF is a function that describes how light is reflected from a given surface based on the incoming and outgoing light directions. To make it easier and more efficient to use BRDF data, it is often organized into various parametric numerical models, known as BRDF models.

This article will introduce the basics of BRDF and classify and describe some well-known BRDF models, including their advantages and disadvantages.

2. BRDF Concept

The surface of real-world objects that we see is actually the result of the surrounding environment's light shining on the object's surface, and then some of that light is reflected back into our eyes. BRDF is a function that describes the relationship between the incoming light on a surface and the reflected light. Its mathematical expression is as follows:

$$f_r(p, \omega_i, \omega_o) = \frac{dL_o(p, \omega_o)}{dE_i(p, \omega_i)} = \frac{dL_o(p, \omega_o)}{L_i(p, \omega_i) \cos \theta_i d\omega_i} \quad (1)$$

In this equation, $f_r(p, \omega_i, \omega_o)$ represents the BRDF; p is a point on the surface of the object, ω_i is the incoming light direction, and ω_o is the observer direction or the direction of the reflected light. $dL_o(p, \omega_o)$ is the differential reflected radiance of the surface in the direction ω_o , and $dE_i(p, \omega_i)$ is the differential illuminance of the surface from the incoming light direction ω_i . θ_i is the angle between the incoming light direction ω_i and the surface normal at shading point p .

By integrating over the entire semi-spherical surface:

$$L_o(p, \omega_o) = \int_{\Omega} f_r(p, \omega_i, \omega_o) L_i(p, \omega_i) \cos \theta_i d\omega_i \quad (2)$$

In general, BRDF describes the distribution of outgoing light after it is reflected from a point on the object's surface, based on the incoming light direction. In computer graphics, BRDF is depicted as an RGB vector, with each of the three components having its own $f_r(p, \omega_i, \omega_o)$ function. Meanwhile, BRDF has three properties: reversibility, energy conservation, and linearity.

BRDF's reversibility is based on the Helmholtz Reciprocity Principle, which indicates that exchanging the incoming and reflected light will not change the value of BRDF:

$$f_r(p, \omega_i, \omega_o) = f_r(p, \omega_o, \omega_i) \quad (3)$$

BRDF is also subject to the principle of energy conservation. The energy conservation equation is as follows (Q represents the corresponding energy):

$$Q_{incoming} = Q_{reflected} + Q_{absorb} + Q_{transmitted} \quad (4)$$

Therefore, it can be inferred that:

$$Q_{reflected} \leq Q_{incoming} \quad (5)$$

Thus, BRDF must satisfy the following integral inequality regarding the energy conservation property:

$$\forall \omega_i, \int_{\Omega} f(p, \omega_i, \omega_o) \cos \theta_i d\omega_i \leq 1 \quad (6)$$

The linearity property of BRDF refers to the fact that, typically, the reflection properties of materials require the calculation of multiple BRDFs. The total reflected radiance of a point on a surface can be simply represented as the sum of the individual BRDF reflected radiances. For example, glossy diffuse reflection can be achieved through the calculation of multiple BRDFs.

3. BRDF Models

In order to make it easier and more efficient to use BRDF data, researchers have tried to organize BRDF into various parametric numerical models. In general, these models can be broadly classified into two types: empirical models and physically-based models. Additionally, if it is possible to directly acquire and use the measured BRDF data in the rendering process, it can achieve a high degree of realism. BRDF models based on this method are called data-driven models.

3.1. Empirical Models

Empirical models are often developed by fitting a mathematical function to a set of data and are used to approximate the behavior of certain phenomena. Empirical BRDF models typically provide simple formulas for fast calculation of reflected light. Due to their efficiency, empirical models are widely used in practice. However, empirical models may not be physically accurate and may not satisfy certain physical laws, such as reversibility and energy conservation.

In 1975, Bui Tuong Phong [1] proposed the Phong Reflection Model, a well-known reflectance model for simulating specular reflections. It was one of the oldest BRDF models and it describes the light reflected by a point on an object's surface to the observer's direction as the sum of several light intensities, its mathematical expression is as follows:

$$f_r(x, \omega_i, \omega_o) = k_d \frac{1}{\pi} + k_s \frac{n+2}{2\pi} [\max(0, \cos \alpha)]^n \quad (7)$$

In the expression, α is the angle between the direction of emitted light and the ideal direction of specular reflection for the incoming light; k_d is the diffuse reflectivity, which is the proportion of energy that is scattered when it comes into contact with the surface of an object; k_s is the specular

reflectivity, which is the proportion of energy that is reflected perpendicularly when it makes contact with the object's surface; n is the specular exponent, which is used to simulate the roughness of the surface. The value of n is proportional to the smoothness of the object's surface.

The Phong model has the advantage of being relatively simple to implement and computationally efficient, and also fits the processing power of the computer at the time. It allows for adjustment of the range and sharpness of specular highlights, depending on the value of the specular exponent parameter n . However, the Phong model can produce unrealistic highlights that are too small or too large, and it does not accurately model the way that light is scattered by rough surfaces. Additionally, the Phong model does not take into account the direction of the observer relative to the surface, which can lead to inaccuracies in the rendered image.

In 1977, Jim Blinn [2] proposed the Blinn-Phong model, which is an improvement over the Phong model in that it simplifies the computation of the specular term. Instead of using the angle between the ideal reflection vector and the view vector, it uses the angle between the half-vector and the view vector. This simplification allows for more efficient computation while still producing similar results to the Phong model. Additionally, the Blinn-Phong model can produce more realistic specular highlights on surfaces with a high specular exponent, as the highlights appear to "spread out" more. However, the Blinn-Phong model can still produce unrealistic highlights if the specular exponent parameter is set too high, and it also does not take into account the direction of the observer relative to the surface.

In 1992, Ward [3] proposed the Ward BRDF model, which is an empirical model that was developed by measuring and fitting the reflectance data of objects. The model is computationally efficient, can be efficiently calculated using the importance sampling method for Monte Carlo integration, and has straightforward and clear parameters for controlling the specular component. It only has three parameters, one controlling the degree of specularity and the other two controlling the roughness of the object's surface. The specific formula is as follows:

$$f_r(x, \omega_i, \omega_o) = \frac{\rho_d}{\pi} + \frac{\rho_s}{\pi\alpha\beta} \cdot \exp \left[-\tan^2 \theta_h \left(\frac{\cos^2 \phi_h}{\alpha^2} + \frac{\sin^2 \phi_h}{\beta^2} \right) \right] \cdot \frac{1}{4\sqrt{\cos \theta_i \cos \theta_o}} \quad (8)$$

The formula includes ρ_d , which represents the base color of diffuse reflection under specular reflection; ρ_s , which represents the specular reflectivity, or the proportion of energy perpendicular to the object's surface that is reflected by the surface; α and β , which are parameters that control the roughness of the object's surface; θ_i and θ_o , which are the zenith angles of the incoming and outgoing light direction, respectively; θ_h and ϕ_h , which are the zenith and azimuthal angles of the halfway vector between the incoming and outgoing light direction.

However, the original Ward BRDF model has energy loss when light scatters. In 2006, Dür [4] proposed an improved normalization factor and presented the following formula to address this issue:

$$f_r(x, \omega_i, \omega_o) = \frac{\rho_d}{\pi} + \frac{\rho_s}{\pi\alpha\beta} \cdot \exp \left[-\tan^2 \theta_h \left(\frac{\cos^2 \phi_h}{\alpha^2} + \frac{\sin^2 \phi_h}{\beta^2} \right) \right] \cdot \frac{1}{4 \cos \theta_i \cos \theta_o} \quad (9)$$

However, the Ward-Dür BRDF model is not always energy conservation. In 2010, Geisler - Moroder and Dür [5] improved the Ward-Dür BRDF model to conserve energy at all angles. This version of the model addresses the issue of energy conservation by introducing a bounded albedo parameter, which helps to ensure that the model remains physically plausible across all angles. The formula for this model is as follows:

$$f_r(x, \omega_i, \omega_o) = \frac{\rho_d}{\pi} + \frac{\rho_s}{\pi\alpha\beta} \cdot \exp \left[-\tan^2 \theta_h \left(\frac{\cos^2 \phi_h}{\alpha^2} + \frac{\sin^2 \phi_h}{\beta^2} \right) \right] \cdot \frac{2[1 + \cos \theta_i \cos \theta_o + \sin \theta_i \cos \theta_o \cos(\phi_o - \phi_i)]}{(\cos \theta_i + \cos \theta_o)^4} \quad (10)$$

The Ward model, despite its advantages in terms of computational efficiency and intuitive parameters, has some limitations. One of the main limitations is that it may not be suitable for all types of materials or situations, and it may not be able to accurately capture certain physical phenomena such as the Fresnel effect, which makes it not as accurate as other models such as the microfacet model that were developed later. Additionally, it may require more computational resources than simpler models.

In 2000, Ashikhmin and Shirley [6] proposed a new BRDF model, which was inspired by the models of Ward [3], Schlick [7], and Neumann et al [8]. The formula for the Ashikhmin-Shirley model is as follows:

$$f_r(x, \omega_i, \omega_o) = f_{r,d}(x, \omega_i, \omega_o) + f_{r,s}(x, \omega_i, \omega_o) \quad (11)$$

$$f_{r,d}(x, \omega_i, \omega_o) = \frac{28R_d}{23\pi} (1 - R_s) \left[1 - \left(1 - \frac{\cos \theta_i}{2} \right)^5 \right] \left[1 - \left(1 - \frac{\cos \theta_o}{2} \right)^5 \right] \quad (12)$$

$$f_{r,s}(x, \omega_i, \omega_o) = \frac{D(h)F(\omega_i)}{4 \cos \langle h, \omega_o \rangle \max(\cos \theta_i, \cos \theta_o)} \quad (13)$$

$$D(h) = \frac{\sqrt{(n_u + 1)(n_v + 1)}}{2\pi} (\cos \theta_h)^{n_u \cos^2 \phi_h + n_v \sin^2 \phi_h} \quad (14)$$

$$F(\omega_i) = R_s + (1 - R_s)(1 - \cos \langle h, \omega_i \rangle)^5 \quad (15)$$

In the formula, R_d is the base color of diffuse reflection under specular reflection; R_s is the specular reflectance, that is, the proportion of energy perpendicular to the object surface that is reflected by the specular reflection; n_u and n_v are parameters that control the degree of specular reflection on the object surface, similar to the Phong model; h is the halfway vector between the light incidence direction ω_i and the outgoing direction ω_o ; $D(h)$ is the probability density function of the halfway vector h ; $F(\omega_i)$ is an approximation of the Fresnel term, also known as Schlick's approximation.

The formula for generating the halfway vector h and the corresponding probability of the outgoing direction using the importance sampling method based on $D(h)$ is as follows, where ξ_1 and ξ_2 are two random variables with independent and identical distributions:

$$\xi_1, \xi_2 \sim U[0,1] \quad (16)$$

$$\phi_h = \arctan \left(\sqrt{\frac{n_u + 1}{n_v + 1}} \tan \frac{\pi \xi_1}{2} \right) \quad (17)$$

$$\theta_h = \arccos \left[(1 - \xi_2)^{\frac{1}{n_u \cos^2 \phi_h + n_v \sin^2 \phi_h + 1}} \right] \quad (18)$$

The Ashikhmin-Shirley model is a new empirical model that has many desirable properties, such as energy conservation and reciprocity. It allows for anisotropic reflection, enabling the creation of effects such as the streaky appearance of brushed metal. The parameters of the model are intuitive, and it takes into account the Fresnel effect, which causes specular reflection to increase as the angle of incidence decreases. Additionally, the diffuse term is not constant, so the diffuse component decreases as the incidence angle decreases. This model is well-suited for Monte Carlo rendering techniques.

3.2. Physically-based Models

Physically-based BRDF models are a class of models used in computer graphics to simulate the way light interacts with surfaces. These models are based on the physical properties of the materials they aim to simulate, such as their microstructure, refractive index, and roughness. However, a physically-based BRDF model must satisfy at least the following two BRDF characteristics: energy conservation, and Helmholtz reciprocity.

In 1967, Torrance and Sparrow [9] derived a rough surface specular reflection model using radiance theory and microfacet theory in their paper. In 1981, Cook and Torrance [10] proposed the Cook-Torrance model, which is an applied version of the Torrance-Sparrow model, and they introduced it into the field of computer graphics. The contribution of Cook and Torrance is that they pointed out that only the microfacets in the midline direction of the reflection direction facing the view

direction have a contribution to the surface reflection term, which simplifies the model and improves the usability of the model. The BRDF defined by the Cook-Torrance model is as follows:

$$f_r(p, \omega_i, \omega_o) = \frac{F(\omega_i, h)G(\omega_i, \omega_o, h)D(h)}{4|\omega_i \cdot n||\omega_o \cdot n|} \quad (19)$$

In the equation, n represents the normal of the macroscopic surface; h represents the halfway vector between the incoming light direction ω_i and the outgoing light direction ω_o , and is the normal of the microsurface; $F(\omega_i, h)$ is the Fresnel term; $G(\omega_i, \omega_o, h)$ is the shadowing-masking function, which is the coefficient of light intensity reduction due to the occlusion of the microspheres; $D(h)$ is the microfacet distribution function, which determines the proportion of microsurface normals oriented towards h .

The Cook-Torrance model is a widely used microfacet model in computer graphics for simulating the appearance of materials such as metal and glass. It has several advantages, including its ability to accurately model the Fresnel effect, which is the way that light reflects off surfaces at grazing angles. It also allows for anisotropic reflection, which can be used to create the appearance of brushed metal. Additionally, it can be used to model a wide range of materials, from dull to glossy surfaces.

One of the main disadvantages of the Cook-Torrance model is that it can be computationally expensive, as it requires the calculation of multiple complex equations. Additionally, it may not be suitable for all types of materials or situations, such as highly rough surfaces or very thin layers. It also requires a large number of parameters to be adjusted to achieve a good fit for measured data.

In 1991, He et al [11]. published a paper proposing a more complex and fully physical BRDF model based on wave optics. This model takes into account the polarization, diffraction, interference, surface conductivity, and roughness of smaller scattering angles of light, and can simulate more optical phenomena than microfacet models, but the computational cost is much higher.

The Oren-Nayar model is a BRDF model for rough surfaces developed by Oren and Nayar [12] in 1994. It is an extension of Lambert's model [13] and accounts for the effects of surface roughness on the way light reflects off a surface. Lambert's model is a simulation of lighting in ideal conditions, and cannot accurately reflect the lighting effects on the surface of objects, especially rough objects. The Oren-Nayar model improves upon this, focusing on modeling rough surfaces, such as gypsum, sandstone, ceramics, etc. It uses a series of microfacets based on Lambert's model and considers the mutual occlusion (shadowing and masking) and mutual reflection lighting between the microfacets. One of the main advantages of the Oren-Nayar model is its ability to simulate the surface roughness of real objects to a certain extent, making the object more textured. However, it can be computationally expensive and may not be suitable for real-time applications.

3.3. Data-driven Models

Data-driven models are a type of general model that provides a way to measure anisotropic BRDFs based on measured data. They record a large set of BRDF materials as high-dimensional vectors and then use dimensionality reduction to calculate a low-dimensional model from these data. This allows for a lookup table-based approach to directly find rendering results, saving a lot of real-time computation. Matusik et al [14]. described how they implemented a series of works and obtained a data-driven reflectance model in their 2003 paper. Additionally, many laboratories have used various instruments to measure the reflectance data of various real-world materials under different lighting angles and observation angles and recorded them in databases that are publicly available, such as the MERL BRDF Database.

Because data-driven models are based on measurements of real-world materials, the resulting renderings are highly realistic, which is one of the main advantages of these models. However, a major drawback is the lack of parameters for adjusting the effects, so it is not possible to modify the data to achieve the desired results. Additionally, data collection is difficult for some extreme angles due to instrument limitations. These models also require a large amount of data and are computationally expensive, making them less suitable for real-time applications such as video games, but they are

suitable for offline rendering such as in movies. They can also be used in graphics research to evaluate the realism of other BRDF models.

4. Conclusion

In conclusion, the BRDF is critical in describing the interactions between light and objects in computer graphics. Various BRDF models have been suggested to make it easier and more efficient to use BRDF data, including empirical models and physically based models. Empirical models often provide simple formulas for fast calculation of reflected light and are widely used in practice. However, they may not be physically accurate and may not satisfy certain physical laws. Physically based models, on the other hand, aim to mimic how light behaves physically on a material as accurately as possible but can be computationally expensive. Data-driven models, which use measured BRDF data to achieve a high degree of realism, are also an option but may not be suitable for real-time applications. In summary, each BRDF model has its own advantages and disadvantages, and the application's particular requirements determine which model should be used.

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