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A new digital image shifting method by considering the geometry of camera sensors

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Abstract. Digital image shifting occurs when the pixels needs to be moved, the algorithm behind this is interpolation. Current digital image shifting algorithm utilizing interpolation has a limitation: it treats all images as the same type of matrix and does not consider how the images are formed – this could cause problems in some sensitive imaging scenarios such as microcopy and precision optical metrology. The goal of our research is to propose a new approach to implement the digital image shifting method. We based our new method off of digital camera sensors, specifically the CMOS sensor. When light (a continuous signal) hits the CMOS sensor, the sensor transforms the light into a discrete signal and its energy into electric energy. Then, this discrete signal then generates the pixels of the image. Our algorithm is similar: The signal is convoluted with the rectangular function, generating a continuous signal, and then we multiply the discrete dirac comb function to generate a discrete signal. After that, we only have to manipulate the discrete dirac comb function to obtain the desired result. Our conducted simulations on the computer proves our method correct, hopefully more experiments can be done in the industrial setting.

Keywords: Interpolation, image shifting, camera, sensor-array.

1. Introduction

Interpolation [1] is the method of estimating new data points by referring to known data points in a discrete set of values. Interpolation plays an important role in scientific data processing – it enables us to obtain high-resolution data points from existing ones. There are several approaches to implement interpolation, such as linear interpolation [2], polynomial interpolation [3], spline interpolation [4], and mimetic interpolation [5].

In digital image processing, interpolation is heavily used to up-sample low-resolution images [6] to match the requirement of the number of pixels in various display settings. Another important application is image shifting [7] for example in object tracking, we need to calculate the displacement of a moving object, and this can be done by displace one of the two images captured at two successive time stamps. Interpolation allows us to perfectly restore the sampled continuous signal (only apply to bandlimited signals) such that we can shift the image with sub-pixel accuracy, which is crucial in many sensitive scientific experiments.

However, current image shifting methods (or we can say the current implementation methods of interpolation in image shifting applications) do not consider the real imaging scenarios: they treat all images as the same type of matrix and do not care how the images are obtained. For example, in several microscopic imaging systems, shifting a target towards the edge of the field-of-view would result in the change of the light rays, and thus the shifted image will have slightly different contrast compared to the unshifted image from the center of the field-of-view. Here in this project, we will explain such scenarios in detail, and we will refine current image shifting algorithms by referring to the configuration of camera sensors.

2. Principle

2.1. Digital image shifting using interpolation

Consider a one-dimensional case: the red points in Figure 1 can be viewed as a 1-D image and we can assume they are sampled from an unknown continuous function $f(x)$. Now the question is if we shift this 1-D image by 0.2 pixels, what are the new values? One way to do this is using interpolation – it provides a means of estimating the function at intermediate points, such that we can obtain values at $x = 0.2, 1.2, \dots, 6.2$. Figure 1(b) shows the estimated function using polynomial interpolation.

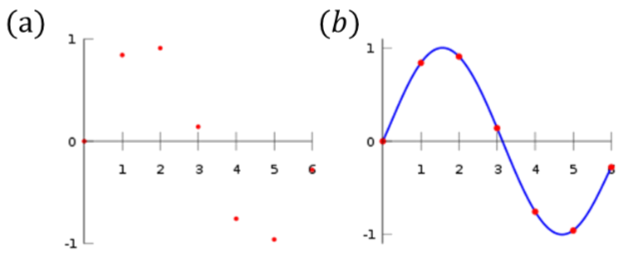

Figure 1. (a) Red dots are the data points of the 1-D image. (b) Blue curve is the interpolated function using polynomial interpolation. Image Courtesy: https://en.wikipedia.org/wiki/Interpolation

2.2. Fresnel equations

The Fresnel equations describe the reflection and transmission of light when incident on an interface between different optical media. [8] It has two sets of equations targeting on s and p polarizations – in both scenarios, the transmittance and reflectance are dependent on the incident angle. An example is shown in Figure 2.

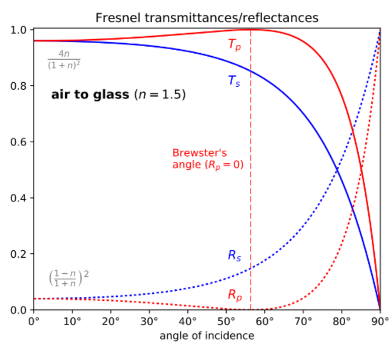

Figure 2. Transmittance (T) and reflectance (R) as a function of the incident angle for light incident from air to glass. S and P indicate the two different polarization states. Image Courtesy: https://en.wikipedia.org/wiki/Fresnel_equations#cite_note-2

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2.3. Shifting the object point reduces the light collection efficiency via camera sensors

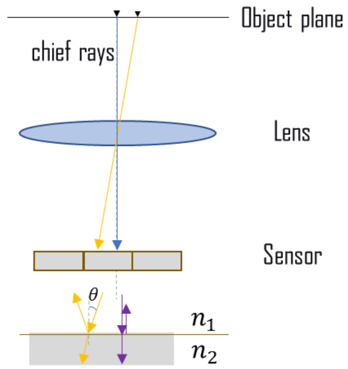

Figure 3. Illustration of the chief rays of the original object point and after shifting.

In many cases, image shifting is caused by shifting the object - this will slightly change the angle of the chief rays. According to Fresnel equations, the light collection efficiency will drop.

3. Method

For simplicity, we suppose the camera is a line-camera (i.e., the sensor array is placed only along one dimension).

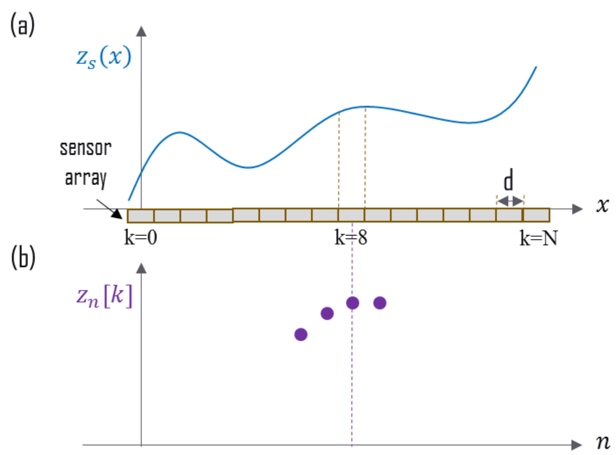

Figure 4. Schematic diagram of digital image formation in camera or camera-based imaging system. In (a), $z_s(x)$ (the blue curve) describes the optical field along x; the gray boxes represent the camera sensor array, and each pixel has a width of d; the total number of pixels is N. In (b), $z_n[k]$ (the purple dots) represents the digital image, which is formed and sampled via the sensor array.

Suppose the optical field in front of the sensor array is $z_s(x)$ (it's a continuous signal). Upon reaching the sensor array, $z_s(x)$ is sampled to discrete signal and the optical energy is transformed to electrical energy. Let's denote the sampled signal (i.e., the image) as $z_n[k]$, then $z_n[k]$ can be written as:

$$z_n[k] = \eta \int_{k-\frac{d}{2}}^{k+\frac{d}{2}} z_s(x)\, dx = \eta \int z_s(x)\, rect(\frac{x-kd}{d})dx\,, \quad k = 0,1,2,\ldots \mathrm{N} \tag{1}$$

Here, η is the energy transformation efficiency from optical to electrical signal. The integration is used to describe the light collection process at each pixel of size d, and we use the rectangular function to assume a flat integration process.

Equation (1) can also be written as:

$$z_n[k] = \eta \left[z_s(x) \otimes rect\left(\frac{x}{d}\right) \right] \left(\sum_{k=0}^{k=N} \delta[n-kd]\right) \quad (2)$$

Here, $\otimes$ denotes convolution and $\sum_k \delta[n-kd]$ is the discrete Dirac comb function. Illustration of Equation (2) is plotted in Figure 2.

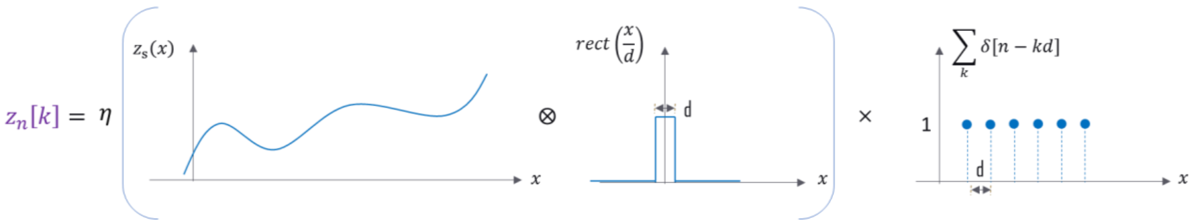

Figure 5. Illustration of Equation (2). $z_n[k]$ can be obtained in two steps: 1. Get the convolution between $z_s(x)$ and $rect(\frac{x}{d})$, which can be viewed as a filtered signal of $z_s(x)$ 2. Sample the convoluted signal from Step #1, using the discrete Dirac comb function.

Equation (2) also suggests that $z_n[k]$ can be viewed as the ideal sampled signal of a new continues signal $\eta \left[z_s(x) \otimes rect\left(\frac{x}{d}\right) \right]$, as shown in Figure 3.

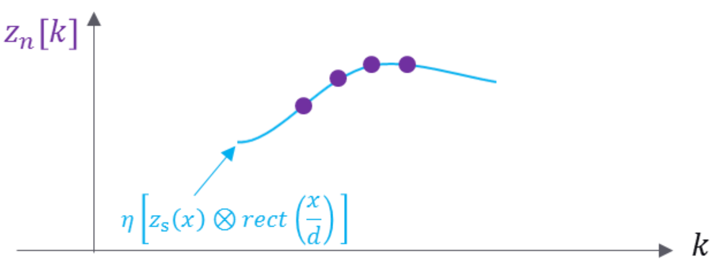

Figure 6. Plot showing that $z_n[k]$ can be viewed as ideal sampling of a new signal $\eta \left[z_s(x) \otimes rect\left(\frac{x}{d}\right) \right]$.

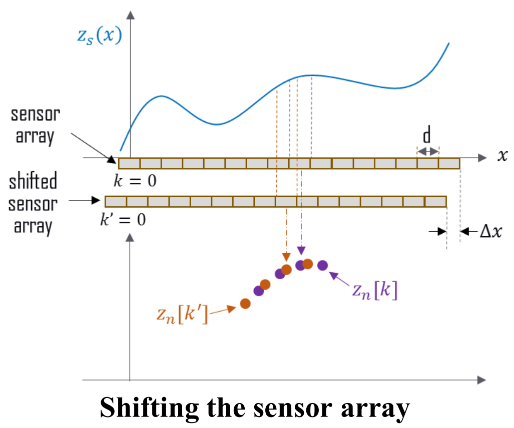

Figure 7. Illustration of the (shifted) digital image formed by shifting the sensor array. The original image is denoted as $z_n[k]$ and the shifted image is denoted as $z_n[k']$. The shifting distance is Δx and we assume it's smaller than 1 pixel (i.e., $\Delta x < d$).

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Now we consider the operation of shifting image in digital image processing. Shifting image corresponds to two real scenarios: 1. shifting the sensor array 2. shifting the object which forms the image. Here, we only consider sub-pixel shifts, as shifting the image by integer pixels corresponds to simple integer shifting of the sampling points.

Here, the optical field does not move (see Figure 3), so Equation (1) and (2) can be written as:

$$z_n[k'] = \eta \int_{k-\Delta x-\frac{d}{2}}^{k-\Delta x+\frac{d}{2}} z_s(x)\, dx = \eta \int z_s(x)\, rect(\frac{x-kd-\Delta x}{d}) dx\,, \;\; k = 0,1,2,\ldots \mathrm{N} \quad (3)$$

$$z_n[k'] = \eta \left[z_s(x) \otimes rect\left(\frac{x}{d}\right) \right] \left(\sum_{k=0}^{k=N} \delta[n - kd - \Delta x]\right) \quad (4)$$

Comparing Equation (2) and Equation (3), we can see the only difference is the Dirac comb function – the one in Equation (3) is simply a shifted version of that in Equation (2). As a result, we can get $z_n[k']$ from $z_n[k]$ using the common image shifting method: firstly, run interpolation to get an estimate of $\eta \left[z_s(x) \otimes rect\left(\frac{x}{d}\right) \right]$, then we sample this signal at the new locations indicated by the new Dirac comb function.

Shifting the object

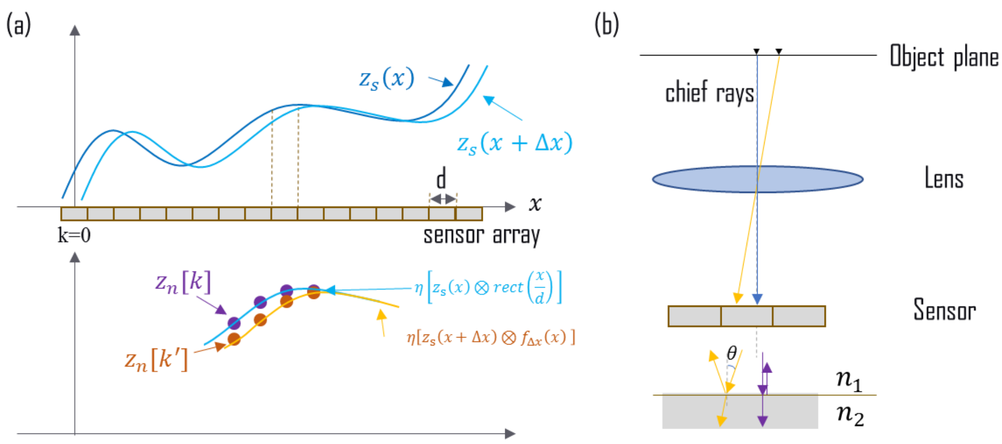

Figure 8. (a) Illustration of the (shifted) digital image formed by shifting the object. $z_s(x + \Delta x)$ denotes the optical field after displacing the object. The original image is denoted as $z_n[k]$ and the shifted image is denoted as $z_n[k']$. The shifting distance is Δx and we assume it's smaller than 1 pixel (i.e., $\Delta x < d$). (b) Schematic diagram of the simplest image formation process (i.e., via single lens): the purple lines represent the light rays from the original object point and the orange lines represent the light rays from the shifted object point. In the bottom figure, n_2 is the refractive index of the sensor material (e.g., for GaAs, $n_2 = 3.97$ @ 580 nm), and n_1 is the refractive index of the top material (air or glass).

In this scenario, the optical field in front of the sensor array (i.e., $z_s(x)$) shifts (see Figure 5(a)). Now write Equation (1) and (2) become:

$$z_n[k'] = \eta \int_{k-\frac{d}{2}}^{k+\frac{d}{2}} z_s(x+\Delta x)\, f_{\Delta x}(x) dx = \eta \int z_s(x)\, f_{\Delta x}(\frac{x-kd}{d}) dx\,, \;\; k = 0,1,2,\ldots,\mathrm{N} \quad (5)$$

$$z_n[k'] = \eta \left[z_s(x+\Delta x) \otimes f_{\Delta x}\left(\frac{x}{d}\right) \right] \sum_k \delta[n - kd]\,, \;\; k = 0,1,2,\ldots,\mathrm{N} \quad (6)$$

Figure 5(a) shows the comparison between the original image $z_n[k]$ and the shifted image $z_n[k']$, from which we can see that while their sampling locations are the same (this is because the sensor array is fixed), the continuous signals they try to sample are different.

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Now let's compare the two shifting image scenarios and see what's in common and what's different. Firstly, we re-write Equation (6) as:

$$z_n[k'] = \eta \left[z_s(x+\Delta x) \otimes f_{\Delta x}\left(\frac{x}{d}\right)\right] \sum_k \delta[n-kd] = \eta \left[z_s(x) \otimes f_{\Delta x}\left(\frac{x}{d}\right)\right] \sum_k \delta[n-kd-\Delta x],\ k = 0,1,2,\ldots,\mathrm{N} \tag{7}$$

By comparing Equation (7) with Equation (4) from scenario #1, we can see that the only difference is that here $z_s(x)$ convolves with a new function $f_{\Delta x}\left(\frac{x}{d}\right)$ instead of the rectangular function $rect\left(\frac{x}{d}\right)$. The reason that we use a different function here can be explained by Figure 5(b): for simplicity, let's consider the central pixel that lies on the optical axis. After shifting the object, we can see that the light rays (orange lines) now hit the sensor with a bigger angle of incidence than the original light rays (purple lines). As a result, the optical energy absorbed by the sensor will be smaller – this can be concluded from Fresnel equations for reflection and refraction. Ideally or by approximation, we can still model $f_{\Delta x}(\frac{x}{d})$ as a rectangular function:

$$f_{\Delta x}\left(\frac{x}{d}\right) \approx a\, rect\left(\frac{x}{d}\right), a < 1, \tag{8}$$

but since the energy absorbed by the sensor gets smaller, the rectangular function here will have smaller amplitude than the one in Equation (2) and Equation (4). In theory, depends on the shifting distance as it is related to the angle of incidence (see Figure 5(b)), but in real imaging conditions, the dependence can be very weak and thus can be neglected.

So, in this scenario, we cannot use the common digital image shifting method to get $z_n[k']$ from $z_n[k]$, which has been briefly discussed in the previous section. Instead, we apply the following steps: 1. Use common image interpolation method to get $Q(x) = \eta \left[z_s(x) \otimes rect\left(\frac{x}{d}\right)\right]$ from $z_n[k]$ (see Equation (2)). 2. Combine Equation (2) and Equation (7) to get $Q_s(x) = \eta \left[z_s(x) \otimes f_{\Delta x}\left(\frac{x}{d}\right)\right]$, which can be obtained in two steps – firstly, apply deconvolution to $Q(x)$ with respect to $rect\left(\frac{x}{d}\right)$ to get $\eta\, z_s(x)$; then convolve $\eta\, z_s(x)$ with $f_{\Delta x}\left(\frac{x}{d}\right)$. 3. Use ideal digital sampling to get $z_n[k']$.

4. Simulation results

Here we will give an example on how to apply the new image shifting method. Consider the discrete signal of $z_n[k]$ in Figure 6(a): firstly, we use polynomial interpolation to get the interpolate signal $Q(x)$ – here since $Q(x)$ is a continuous signal we will use more sampling points to represent it (~10× than $z_n[k]$). Then, we apply deconvolution to $Q(x)$ using the rectangular function $rect\left(\frac{x}{d}\right)$ to get the intermediate result $\eta\, z_s(x)$ (see Figure 6(b)), which then convolves with $f_{\Delta x}\left(\frac{x}{d}\right)$ to get $Q_s(x)$ (see Figure 6(c)). Finally, we sample the new $Q(x)$ at shifted locations to get $z_n[k']$. Here, for simplicity, we set $d = 1$ and $f_{\Delta x}\left(\frac{x}{d}\right) = 0.8\, rect(\frac{x}{d})$.

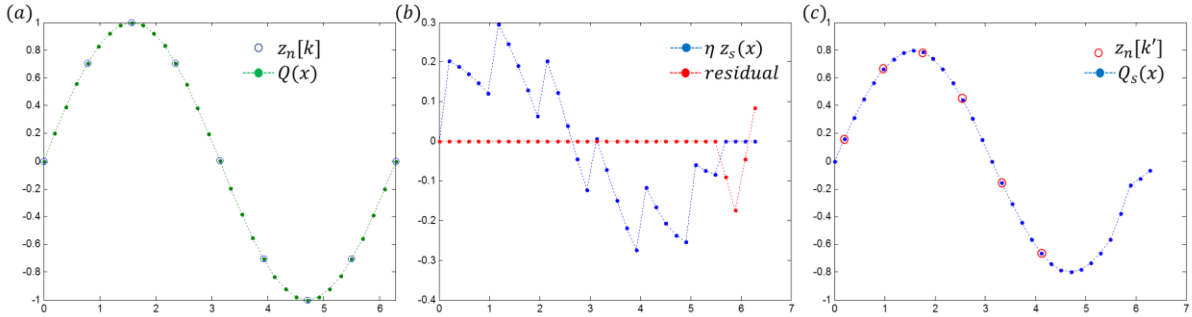

Figure 9. Simulation results of the algorithm summarized in the last paragraph of Method section.

From the results we can see that our method can get the expected shifted signal with very good accuracy, except for the tails near the end of the signal. The corrupted signal at the tails are caused by the deconvolution [9] process – as shown in Figure 9(b), deconvolution will generate some residuals at the end of the signal. Other than this, the shifted signal (red circles in Figure 9(c)) follows the sine waveform very well. At last, it's worth mentioning that the amplitude of the shifted signal is reduced to 0.8 as expected – this is the consequence of using the new rectangular function to incorporate the drop in optical collection efficiency towards the edge of the field of view.

5. Discussion

In the "Shifting the object" session of Method, we justified that in theory our new interpolation method could provide a more accurate shifted image compared to current digital image shifting methods. To prove this, however, we need to run real imaging experiments to compare the calculated shifted image to the real shifted image collected from the camera. Considering that we are working on sub-pixel shifting and the typical pixel size of camera sensors are $5{\sim}20\mu m$, it's better to run the experiments on a microscopy system – a microscope provides us the necessary accuracy to perform the shifting in the micron level. The experiment should be conducted in the following steps: 1. Get a sample with enough small features (e.g., a chip or a standard calibration plate) and grab an image (let's call it I_0) using the microscope. 2. Shifting the sample using the precision stage by a sub-pixel distance and grab a new image (let's call it I_s). 3. Generate the calculated shifted image from I_0 using the common method and our proposed new method, and then compare the calculated images with I_s – the difference between I_s and the calculated images can serve as a metric to tell which method gives better result. This experiment will be one of the future works related to our project.

Although in theory our proposed method could provide better results than current algorithm, it still has one potential issue related to deconvolution. From the signal theory we know that in some cases, deconvolution does not have a solid solution and we can only get an estimated result with minimal error. For these cases, we need to run more simulation tests to figure out how big the error is and whether it will render the new method ineffective. Another problem is how to correctly pick the new rectangular function based on the geometry of the sensors and the performance of the optical system. In the simulation section, we use the simplest form – a rectangular function with reduced amplitude. However, in reality, it might be necessary to derive a completely new function based on the calibration results of the optical system.

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