

The effect of mild to severe illness on COVID-19 based on improved SIR modeling

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Abstract. COVID-19, as a worldwide epidemic, has led to a wide range of infected individuals with negative implications for the economy, travel, and governance. Previous studies have modeled the transmission mechanism of COVID-19 based on the SIR model and its variants, where populations are categorized as susceptible, infected, and recovered. However, in clinical settings, infected individuals are further subdivided into mildly infected and severely infected individuals, and these severely infected individuals often carry viruses that are more infectious and pathogenic. To tackle the above issues, we propose a novel SI-2. We theoretically derive the disease-free equilibrium of the proposed dynamical system, as well as the basic reproduction number. The impact of the infection rate, recovery rate and mortality rate of mildly and severely infected patients on controlling the spread of the epidemic is further discussed based on the derived basic reproduction number. These results have clinical and guiding implications for the early protection and control of severely infected patients.

Keywords: Mathematical Modeling SI-2, Positive and Negative State Transfer, the Reproduction Number, Guiding Implication

1. Introduction

New Coronavirus (NCCV), or "New Coronavirus", has been a major global epidemic for the past four years, with a devastating impact on global health. As of 12 April 2023, 762,791,152 people worldwide have been infected with NCPV, of which 6,897,025 have died from chronic or acute outcomes of NCP (WHO, 2023), which is as bad as the 1976 Ebola outbreak in southern Sudan and the DRC. The Ebola virus. Over the past three years, the COVID-19 pandemic has had a catastrophic impact on the world, as the decimation of the population and the excessive medical stress caused by the new coronavirus have caused enormous financial and economic trauma to individual countries. As a result, there was a surge in research to study and predict the spread of the virus, leading to the development of various models such as the SIR model and its variants.

Researchers in various countries have studied and applied the disease transmission dynamics model as well as the SIR model, and some researchers have used the basic SIR model to make long-term predictions of the sporadic COVID-19 epidemics in different regions of China, including the end time and final size, the peak and peak time of current confirmed cases, and the cumulative number of cleared cases (Pei, L., Hu, Y. Long-term prediction of the sporadic COVID-19 epidemics induced by delta-virus in China based on a novel non-autonomous delayed SIR model [1]. For example, Exploring Susceptible-Infectious-Recovered (SIR) Model for COVID-19 Investigation [2]. Some papers have also

focused on the incubation period of novel coronaviruses, considered occult transmission populations, constructed improved SEIR-based transmission kinetics models for Neoplastic pneumonia, and made predictions for specific geographical areas, e.g. "Analysis and prediction of Neoplastic pneumonia outbreaks based on improved SEIR models" [3]. There are even many derived model systems, for example, considering measures such as government control and personal protection, a new kinetic model SLIR was proposed based on the classical infectious disease SIR model by introducing low-risk groups [4].

All of these models have largely analyzed and predicted different dimensions of infection in neocolonial, but none of the predictive models proposed so far have focused on the impact of the interchangeability of mild and severe neocolonial pneumonia on the study. In order to investigate the importance of the less severe and more severe patients for outbreak control respectively, this paper redesigns a variant of the disease kinetic model, the SI-2 model, is redesigned from the perspective of disease interchange, suggesting that even in the case of disease interchange, the main impact on the disease-free equilibrium point will be on patients with mild disease, who should also be the focus of government efforts to control the epidemic.

Therefore, the paper is divided into four main sections starting with the introduction section, moving on to the basic introduction of the disease dynamics theory model and the definition and interpretation of the basic variables in this paper, then moving on to the SI-2 model calculations and the analysis of the implications of the SI-2 model findings for practical epidemic control using China as an example, and finally summarizing the main content and findings of the full paper (see Table 1).

Table 1. Parameter description.

Parameter	Interpretation
$b(N)$	Birth rate
μ	Natural mortality rate
μ_1	Mild virus related mortality rate
μ_2	Severe virus related mortality rate
β	Coefficient of transmission for virus transmission
β_{11}	The transmission probability for general public contact with people who are infected by the mild virus and then they become to the patient in mild symptom and carry the mild virus
β_{21}	The transmission probability for general public contact with people who are infected by the severe virus and then they become to the patient in mild symptom and carry the mild virus
β_{22}	The transmission probability for general public contact with people who are infected by the severe virus and then they become to the critical patient and carry the severe virus
γ_1	Cure rate for patients with mild infections
γ_2	Cure rate for patients with severe infections
ν	Probability of minor illness to major illness
λ	Probability of serious illness to minor illness

2. The Model

2.1. Permutation

SIR models have a long history, beginning with Daniel Bernoulli's research on inoculation against smallpox in one of his papers in 1760 [5]. However, it was not until Kermack and McKendrick studied the Black Death epidemic in London in 1927 that the SIR hamlet model was accurately formulated [6]. The model divides the population within the epidemic into three categories: S, Susceptible, those who do not have the disease but lack immunity and are susceptible to infection when in contact with a susceptible person; I, Infective, those who have contracted an infectious disease that can be transmitted to members of the S category; and R, Removal, those who have been quarantined, or have become

immune as a result of recovery from the disease. The different types of persons have their own different rates of infection mortality or cure. A model of infectious disease dynamics was drawn up by analyzing the links between the transitions between different types of members.

In 1932 Kermack and McKendrick went on to develop the SIS model, an epidemiological model that focuses on diseases that have the potential for secondary infection after cure, i.e., a model in which there are only two basic subjects, the S class, Susceptible, and the I class, Infective. They developed the threshold theory of infectious disease dynamics based on their study of these models, which is the identification of the disease-free equilibrium (DFE). Kermack and McKendrick's SIR model is arguably the most classic and fundamental of the infectious disease models and has made a seminal contribution to the study of infectious disease dynamics.

2.2. SI-2 Model

During the epidemic, many institutions and scientists used infectious disease models to predict NCC infections, many of which classified populations as S, I and R and used SIR models to make predictions, while others improved the basic SIR model to take into account the different probabilities of infection following vaccination. However, the previous models did not take into account the fact that the new coronavirus is a mild or severe virus, so I have taken this into account and proposed the SI-2 model. The population is divided into three parts, the susceptible population S, the mildly infected population I_1 , and the severely infected population I_2 , where S is considered to be the general population that has not been infected with neocoronavirus until now and has a certain probability of being infected by other people with neo-coronavirus to become infected with neo-coronavirus. Patients with I_2 severe virus are defined as those who have been infected and are carrying a severe virus. The mortality rates for these three populations are also different, with the S population having the lowest mortality rate as μ , the I_1 population having a higher mortality rate than the S population and a lower mortality rate than the I_2 population, as μ_1 , and the I_2 population having the highest mortality rate, as μ_2 . Thus, in the SI-2 model in Figure 1, we have three observations:

1) Minor and major patients are interchangeable and the probability of transferring a minor to a major illness is not the same as a major to a minor illness

2) There are three different probabilities of being infected with a light or heavy virus in a susceptible population S. There is a β_{11} chance of becoming mildly ill after exposure to a mildly ill person (2-1), a β_{21} chance of becoming mildly ill after exposure to a severely ill person (2-2), and a β_{22} chance of becoming severely ill after exposure to a severely ill person (2-3). chance of becoming seriously ill after exposure to a seriously ill person (2-3).

Exposure to a mildly ill person who becomes seriously ill here should not be seen as a new and separate infection, but rather as a case of exposure to a mildly ill person who first becomes mildly ill, but gradually becomes more seriously ill. This is a "mild-to-severe conversion - first infected with a mild virus" process. Figure 1 shows the SI-2 model in the below:

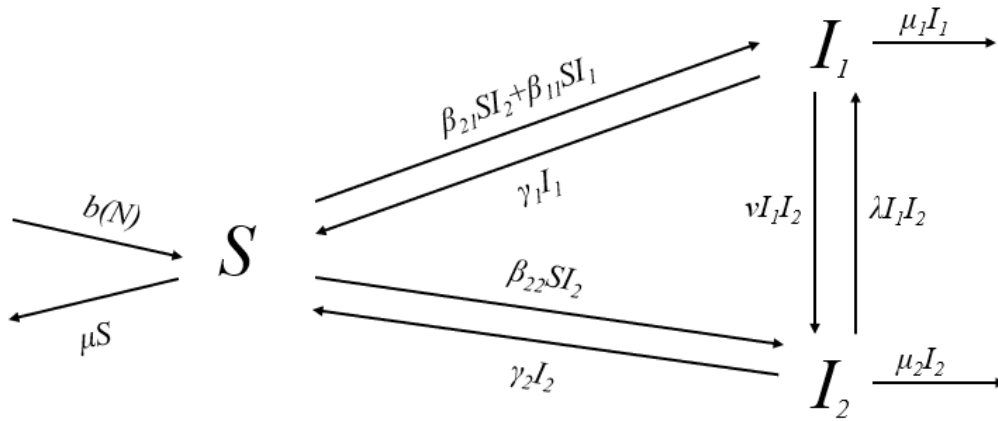


Figure 1. SI-2 model.

Base on the above discussion and the flowchart in Figure 1, the model is formulated as follows:

$$\frac{dI_1}{dt} = \beta_{21}SI_2 + \beta_{11}SI_1 - \gamma_1I_1 - \mu_1I_1 + (\lambda - v)I_1I_2 \quad (1)$$

$$\frac{dI_2}{dt} = \beta_{22}SI_2 - \gamma_2I_2 - \mu_2I_2 + (v - \lambda)I_1I_2 \quad (2)$$

$$\frac{dS}{dt} = b(N) - \mu S + \gamma_1I_1 + \gamma_2I_2 - (\beta_{21}I_2 + \beta_{11}I_1 + \beta_{22}I_2)S \quad (3)$$

Here the I_1 and I_2 are treated as the negative position, only S is treated as the positive position.

3. The Results

3.1. The basic reproduction number

Only the I_1 and I_2 treat as the negative status, I_1 and I_2 can transfer to each other, which progression do not treat as new infections, but rather the progression of an infected individual through the various compartments. Hence,

$$\mathcal{F} = \begin{pmatrix} \beta_{21}SI_2 + \beta_{11}SI_1 \\ \beta_{22}SI_2 \\ 0 \end{pmatrix} \quad (4)$$

$$\mathcal{V} = \begin{pmatrix} \gamma_1I_1 + \mu_1I_1 - (\lambda - v)I_1I_2 \\ \gamma_2I_2 + \mu_2I_2 - (v - \lambda)I_1I_2 \\ -b(N) + \mu S - \gamma_1I_1 - \gamma_2I_2 + \beta_{21}SI_2 + \beta_{11}SI_1 + \beta_{22}SI_2 \end{pmatrix} \quad (5)$$

The infected compartments are I_1 and I_2 . An equilibrium solution with $I_1=I_2=0$ has the form $x_0 = (0, 0, S_0)'$, where S_0 is any positive solution of $b(S_0)=dS_0$. This will be a DFE if and only if $b'(S_0) < d$. Without loss of generality, assume $S_0=1$ thereafter we can get approach to the DEF. Then, the linearization of system at the DFE is given by

$$D\mathcal{F}_{(x_0)} - D\mathcal{V}_{(x_0)} = \begin{pmatrix} F - V & 0 \\ J_1 & J_2 \end{pmatrix} \quad (6),$$

where J_1 and J_2 are 1×2 and 1×0 matrices respectively, and

$$F = \begin{pmatrix} \beta_{11} & \beta_{21} \\ 0 & \beta_{22} \end{pmatrix} \quad (7)$$

$$V = \begin{pmatrix} \gamma_1 + \mu_1 & 0 \\ 0 & \gamma_2 + \mu_2 \end{pmatrix} \quad (8)$$

Since all eigenvalues of J_1 and J_2 have positive real parts, so the stability of x_0 is determined by the eigenvalues of matrix $F - V$. Moreover, all eigenvalues of $F - V$ have negative real parts if and only if the $\rho(FV^{-1}) < 1$, then the generation matrix is formed that the FV^{-1} ,

Giving

$$V^{-1} = \frac{1}{(\gamma_1 + \mu_1)(\gamma_2 + \mu_2)} \begin{pmatrix} \gamma_2 + \mu_2 & 0 \\ 0 & \gamma_1 + \mu_1 \end{pmatrix} \quad (9)$$

That FV^{-1} is,

$$FV^{-1} = \begin{pmatrix} \frac{\beta_{21}}{\gamma_1 + \mu_1} & \frac{\beta_{21}}{\gamma_2 + \mu_2} \\ 0 & \frac{\beta_{22}}{\gamma_2 + \mu_2} \end{pmatrix} \quad (10)$$

Which has eigenvalues

$$\varphi_1 = 0 \quad (11)$$

$$\varphi_2 = \frac{1}{\gamma_1 + \mu_1} \quad (12)$$

Thus, the basic reproduction number can be expressed as

$$R_0 = \frac{1}{\gamma_1 + \mu_1} \quad (13)$$

Therefore, the disease free point can be approached if $R_0 < 1$ and cannot be approached if $R_0 > 1$, where R_0 is defined in above.

3.2. Analysis of the practical implications of the model's conclusions

The cross-infection of new coronary pneumonia with mild and severe disease and the interchangeability of mild and severe disease has always existed and has a high probability. The point of disease-free equilibrium, which is often referred to as the stage of universal immunization (where the number of people cured of the disease in question is much greater than the number of new infections), can only be reached if the sum of these two variables is greater than one, i.e. if the basic reproduction number is less than one.) In order to reach the stage of universal immunization, according to R_0 , we can either increase the cure rate of patients with minor diseases or directly increase the mortality rate of this group of people. However, for each country, the latter approach would undoubtedly lead to a significant decline in population numbers, which would affect the social workforce and even lead to economic decline. Therefore, it is clearly more feasible to increase the cure rate for the mildly ill, which means that the government should primarily control and try to reduce the number of the mildly ill. The human resources, the pressure on the health care system and the financial investment required to control mainly the mildly ill are relatively small, and the cost is lower and the benefit higher, because the two conventional means of control known to countries today are: concentrating the mildly ill in an appropriate hospital, such as the square cabin hospital in China, and treating them together (3-2-1); or isolating them individually at home and taking medication to recover slowly (3-2-2). Both of these methods can be supervised by the government and the relevant medical institutions, but the (3-2-2) method can also be coordinated through the community and the family, so it is less costly in comparison.

This conclusion and preventive measures are also backed up by realistic data and examples, such as in China. China's total GDP has maintained its growth trend for the four years prior to the arrival of the new crown epidemic in 2020. The three-year totals were \$743,583 billion (2016, up 6.7% over the

previous year), \$820,754 billion (2017, up 6.8% over the previous year), \$919,281 billion (2018, up 2.1% over the previous year), and \$990,865 billion (2019, up 6.1%). However, in the first year of the epidemic, 2020, according to the "Statistical bulletin on the development of the national medical insurance system in 2020" given by the National Medical Insurance Administration of China on 20201-6-08[7], China's total national medical insurance expenditure reached a total of 210.32 billion yuan, an increase of 0.9% over the previous year and accounting for about 2.1% of the GDP of that year. In contrast, China's GDP in that year totaled 1,013,567 billion yuan, an increase of only 2.2% over the previous year (article "China's GDP in 2020 finally verified at 1,013,567 billion yuan" by Xinhua News Agency, forwarded by the Central People's Government of the People's Republic of China on 2021-12-17) [8]. This is understandable as, at the beginning of the epidemic, much infrastructure needs to be built, and because of the inexperience of the new virus, a lot of experimental research is necessary, as well as the government's adoption of a uniform quarantine approach that does not differentiate between serious and minor cases of NCC, all leading to a high demand for medical investment. However, in 2021, China continued to use a non-differentiated isolation of patients with severe and mild illnesses, resulting in a further increase in total national health care expenditure to RMB 240.4310 billion, an increase of 14.3% over the previous year (National Development and Reform Commission of the People's Republic of China, Development and Reform Work, Employment and Income, Employment Income Social Security Consumption Topics, 2022-7-26) [9]. This can be seen as an additional burden on the national treasury from the continued adoption of a plan that does not differentiate between segregation and does not segregate priorities. However, 2022 was a new phase in the control of the new crown epidemic, and in that year the government adopted a separate approach to the control of mild and severe cases, mainly for mild cases, and adopted home isolation and community-based centralized isolation as the two main routes (Chinese government website, authoritative release on the State Council's joint prevention and control mechanism, transcript and press conference, 2022-12-07) [10]. As a result, according to the data given by the specialized agencies, the total national health insurance expenditure was RMB 244,3172 million, but only because the expenditure on employee health insurance was elevated in that year, accounting for about two-thirds of the total expenditure, which made the total expenditure rise further, but it can be seen that the spending on new crowns, etc. was much less than in 2021 (Central People's Government of the People's Republic of China, 2022 Medical Security Business Development Statistical Snapshot, News, Government Link, Sectoral Topics, 2023-3-10)[11]. Thus, it is true that in the case of cross-infection with new crowns and interchangeability of mild and severe diseases adopting a tight control and cure approach mainly for mildly ill patients does lead to a more efficient and less laborious way of reaching a disease-free equilibrium point and completing an ideal state of universal immunization.

Therefore, I would say that if governments had taken such preventive and control measures earlier, there would probably have been fewer deaths, less financial losses and a quicker economic recovery. I also very much hope that if new outbreaks of unknown infectious diseases emerge in the future, global experts will try to analyze the problem from this perspective and take control measures.

4. Conclusion

In conclusion, the SI-2 model study on the impact of the interchange of mild and severe disease in the new crown epidemic concludes that controlling the mortality rate of mildly ill patients and increasing the cure rate of mildly ill patients is the key to achieving a disease-free equilibrium point for full immunization and should be the best surveillance approach for governments to adopt to maximize resources and minimize financial expenditure. The key implication of this paper is that this research has led to new policy ideas for controlling transmissible diseases, so that the next time the world needs to respond to an epidemic such as this one, there will be a broader range of ideas to facilitate a faster response to these outbreaks, with minimal human and economic losses.

However, there are some limitations to this research that need to be taken into account. The ideas presented in this paper are based on known viral characteristics and modes of infection, so in order to control unknown infectious diseases more comprehensively, medical identification of the pathology of

the virus and scientific assessment of the impact of environmental factors on infectiousness are required before an optimal solution can be adopted to control the spread of the disease.

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