Proceedings of the 3rd International Conference on Computing Innovation and Applied Physics
DOI: 10.54254/2753-8818/25/20240975

Minimization of transportation costs using linear programming

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Abstract. This paper intricately explores the utilization of Linear Programming (LP), a distinguished mathematical optimization technique, within the logistics and transportation sector, a domain persistently pursuing methods to bolster efficiency and curtail expenses. LP, characterized by its adeptness in optimizing a linear objective function subject to linear constraints, emerges as a pragmatic solution. This exposition elucidates how LP can be harnessed to ascertain the most economically viable transportation routes and quantities of goods transported from warehouses to consumers. A case study is introduced to spotlight the pragmatic applicability of LP in authentic scenarios, and the potential fiscal savings corporations can realize through its adoption. While recognizing the merits of LP, such as clarity, versatility, and computational efficacy, the paper also sheds light on its limitations, involving its dependency on certain presumptions, a necessity for precise data, and its concentration on single-objective optimization. The discourse concludes by prospecting the future of LP in transportation, exploring its amalgamation with artificial intelligence, machine learning, multi-objective optimization, and green logistics, intending to underscore the significance of LP in transportation and furnish insights for ensuing research and application.

Keywords: Linear programming, transportation, python, logistics optimization, cost minimization.

1. Introduction

Every day, people have goods that need to get from factories to stores in many cities. However, factories only have a limited number of trucks, a budget to stick to, and only so many hours in the day. The puzzle of moving these goods efficiently is not new. Centuries ago, traders faced similar challenges, deciding which route to take with their horse carts, considering the road conditions and safety from bandits.

With the industrial revolution and global expansion, these challenges magnified. Trains, planes, and trucks emerged, connecting manufacturers to consumers across vast distances. It was about choosing the quickest route and the most cost-effective one. Enter the world of linear programming (LP), a nifty technique developed to crack such puzzles.

LP is like that brilliant friend who knows the shortest way around town, but for businesses. It helps them allocate their limited resources, like money and time, in the best way possible to get the most out of them. Born during the tough times of World War II, when countries had to move supplies and troops with great precision, LP evolved as a savior to address these challenges.

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An American mathematician, George Dantzig, introduced the Simplex method in 1947[1]. Think of it as a step-by-step guide to navigating the complex web of transportation decisions. While it has been tweaked and polished over the years, its conscience remains unchanged. Using LP, businesses can figure out how much of a product to produce or how to transport goods at the least cost, ensuring they maximize profits.

A good example is Lubcon Limited, a lubricant manufacturer [2]. With their main facility in Ilorin, they have to ship products to various districts, each with a different distance from the headquarters. Every district has a demand that must be met, but distances vary, making the distribution tricky. It is like a jigsaw puzzle, figuring out how to move everything with the least hassle and cost.

However, it is not just about distance. Manufacturers juggle other concerns like capital, workforce, and even regional preferences. LP steps in as a trusted guide, helping businesses like Lubcon ensure their products reach every corner, keeping customers happy and costs in check.

Linear programming (LP) is a powerful mathematical tool for optimizing a linear objective function subject to linear equality and inequality constraints. Its properties, such as clarity, versatility, and efficiency, have made it fundamental in various fields, ranging from economics to engineering. In logistics and transportation, the significance of LP is paramount. Companies can save substantial costs and improve operational efficiency by employing LP. This study aims to highlight the value of LP in transportation, explore its application, and conduct a case analysis to manifest its practical utility.

2. Literature review

Transportation problems are a subset of linear programming problems that deal with the optimal allocation of resources from sources to destinations. They have been widely studied and applied in various fields, such as logistics, economics, engineering, and operations research. The classical transportation problem was first formalized by Monge in 1781 and later studied by Tolstoi, Kantorovich, Hitchcock, Koopmans, and others. The problem involves minimizing the total transportation cost of shipping goods from a set of suppliers to a set of customers, subject to supply and demand constraints [3].

Several extensions and variations of the transportation problem have been proposed to capture the complexity and uncertainty of real-world scenarios. Some of the common extensions include multi-objective transportation problems, fuzzy transportation problems, stochastic transportation problems, multi-modal transportation problems, and green transportation problems [4, 5, 6, 7, 8]. These extensions involve additional objectives or constraints, such as environmental impact, service quality, risk, reliability, and mode choice.

Various methods and techniques have been developed to solve transportation problems and their extensions. The most popular methods include linear programming, integer programming, mixed-integer programming, and network flow algorithms. These methods can handle large-scale problems efficiently and provide optimal or near-optimal solutions. However, they may be unable to cope with non-linearities, uncertainties, or multiple criteria often present in real-world situations. Therefore, some researchers have also explored other approaches, such as heuristic algorithms, meta-heuristics, simulation, artificial intelligence, and hybrid methods. These approaches can offer more flexibility and adaptability, but they may sacrifice optimality or computational efficiency [9, 10, 11, 12].

This paper applies linear programming to minimize transportation costs for a company transporting goods from three warehouses to four customers. The author formulates the problem as a linear programming model and solve it using Python. The paper also analyzes the results and discusses this approach’s benefits and limitations.

3. Linear programming basics

Linear programming (LP) offers a structured approach to solving optimization problems, where both the constraints and the objective function are linear in nature. Let’s delve into the components of LP.

•Decision Variables: These variables represent the unknowns in the problem, typically quantifying the main decisions to be made. In transportation, these could indicate the number of units to transport between different locations.

•Objective Function: This function, either maximized or minimized, describes the main goal of the problem. In our context, this would be to minimize transportation costs.

•Constraints: These are the restrictions or limitations imposed by external factors. For instance, the capacity of a warehouse or the demand of a customer can act as constraints in transportation problems.

4. Application in transportation

The essence of the transportation problem lies in the optimization of costs or time while ensuring goods are transported from one point to another, satisfying certain constraints.

1.Defining the Variables: Typically, xij might represent the number of units shipped from warehouse i to customer j.

2.Setting up the Objective Function: This usually involves minimizing costs. Summing up all individual costs (shipping units xij at cost cij) gives the total cost which needs minimization.

3.Constraints Consideration: This includes:

•Supply Constraints: Each warehouse has a limited supply.

•Demand Constraints: Each customer has a certain demand that must be fulfilled.

4.Solving the Model: Tools such as MATLAB, Python's PuLP library, or specialized software like CPLEX can be employed to find the optimal solution.

5. Case analysis

The practical application of LP can be illustrated by considering a hypothetical transportation problem. Assume we have 3 warehouses (A, B, C) and 3 customers (X, Y, Z). The supply at each warehouse, demand at each customer, and unit transportation costs are as follows:

1. Supply and Demand (Table 1):

Table 1. Supply and Demand

	A	B	C
Supply	30	70	50
Demand	50	50	50

2. Unit transportation cost (cost per unit from warehouse to customer) (Table 2):

Table 2. Unit Transportation Cost

	X	*Y*	*Z*
A	4	6	8
B	2	6	8
C	2	4	6

Upon implementing the linear programming model using Python's PuLP library, a strategic transportation plan was developed, which minimizes the overall transportation cost while satisfying the demands of all customers and respecting the supply constraints of all warehouses. The results yield an optimal transportation cost of 700.0 units.

The computed transportation strategy involves the following decisions:

30 units will be transported from Warehouse A to Customer Z.

50 units from Warehouse B to Customer X.

20 units from Warehouse B to Customer Y.

30 units from Warehouse C to Customer Y.

20 units from Warehouse C to Customer Z.

This configuration doesn't consider transporting any units from Warehouse A to Customers X and Y, from Warehouse B to Customer Z, and from Warehouse C to Customer X, either because it is not cost-efficient or the demands are being met effectively through alternative routes.

The detailed transportation plan can be visualized as follows (Table 3):

Table 3. Result solved by python

	X	*Y*	*Z*
A	0	0	30
B	50	20	0
C	0	30	20

This tabulated format indicates the quantity of units to be transported from each warehouse (rows) to each customer (columns). The numerical values represent the optimal quantities, yielding the lowest transportation cost while satisfying customer demands.

In synthesizing the outcomes, the model has notably considered both the transportation cost matrix and the associated supply and demand constraints to formulate an optimized transportation strategy. The practical implications of this strategic plan warrant potential cost savings and enhanced logistical operations. Further research or iterative analyses could potentially consider additional real-world factors such as seasonal demand variations, logistical disruptions, or varying cost structures to extend the applicability and robustness of the model.

6. Discussion

LP's foundational strength rests on its structured approach to dissecting complex problems, making them solvable through mathematical modeling. It empowers businesses to predict optimal transportation routes and quantities, directly influencing their bottom lines. In the presented case, by merely adjusting the transportation flow based on LP's suggestions, considerable savings were observed.

However, every model is a simplified version of reality. The underlying assumptions in LP—proportionality, additivity, and certainty—can sometimes be at odds with real-world complexities. Our study models transportation costs as linear, which might overlook potential non-linearities due to factors like bulk shipping discounts or dynamic pricing. Likewise, we've assumed static supply and demand scenarios, which can vary based on seasons, market trends, or unforeseen global events.

Moreover, our study's primary focus on cost minimization could unintentionally obscure other equally critical objectives. For instance, while a route may be the cheapest, it might not be the most environmentally friendly or the quickest. And then there's the pivotal concern of data accuracy. LP is only as effective as the data fed into it. Erroneous cost estimations, miscalculated supply numbers, or misjudged demand figures can divert businesses from truly optimal decisions.

The future of LP in transportation lies at the intersection of technological innovation and modeling advancements. As the dynamics of transportation continually evolve, so must the models that aim to optimize it.

One of the most promising avenues is the integration of LP with artificial intelligence (AI) and machine learning (ML). These technologies can enhance predictive analytics, ensuring more adaptive and accurate models that reflect real-time changes. For example, AI could predict fluctuations in demand based on market trends, allowing the LP model to adapt accordingly.

Another exciting prospect is the exploration of multi-objective optimization, where the model doesn't solely focus on cost but balances it with other criteria like delivery times, environmental impact, or route safety. Such an approach would yield solutions that align better with broader business goals and societal responsibilities.

Green logistics also paints an inspiring future trajectory. As environmental concerns gain traction, LP models can be modified to prioritize eco-friendly transportation solutions. This could encompass optimizing routes that reduce carbon footprints, promoting transportation modes with lesser environmental impacts, or even integrating renewable energy sources in transportation.

7. Conclusion

Linear programming (LP) in transportation presents a captivating panorama of efficient decision making, especially concerning cost minimization. Our case study has illuminated the vast potential LP harbors in optimizing resources, and consequently, expenses. However, a comprehensive evaluation necessitates an in-depth look into both its strengths and inherent limitations.

In retrospect, our exploration into the realm of LP in transportation affirms its invaluable role in aiding businesses to make informed, cost-effective decisions. While the model’s inherent limitations underscore the importance of contextual awareness and continuous refinement, the prospective fusion of LP with modern technologies promises a future of even more sophisticated and holistic optimization strategies. As businesses grapple with the multifaceted challenges of the 21st century, tools like LP will undeniably remain at the forefront of strategic resource allocation and decision-making.

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Appendix

Python codes employed to fit the linear programming model for logistic data.

```
import pulp

# Data
costs = {
    (‘A’, ’X’): 4,
    (‘A’, ’Y’): 6,
    (‘A’, ’Z’): 8,
    (‘B’, ’X’): 2,
    (‘B’, ’Y’): 6,
    (‘B’, ’Z’): 8,
    (‘C’, ’X’): 2,
    (‘C’, ’Y’): 4,
    (‘C’, ’Z’): 6
}
supply = {‘A’: 30, ’B’: 70, ’C’: 50}
demand = {‘X’: 50, ’Y’: 50, ’Z’: 50}

# Define Problem
prob = pulp.LpProblem(“TransportationProblem”, pulp.LpMinimize)
```

DOI: 10.54254/2753-8818/25/20240975

```

# Define Decision Variables
x = pulp.LpVariable.dicts(“x”, ((i, j) for i in supply for j in demand), lowBound=0)

# Define Objective Function
prob += pulp.lpSum([x[i, j] * costs[i, j] for i, j in x])

# Define Constraints
for i in supply:
    prob += pulp.lpSum([x[i, j] for j in demand]) <= supply[i], f”SupplyConstraint_{i}”

for j in demand:
    prob += pulp.lpSum([x[i, j] for i in supply]) >= demand[j], f”DemandConstraint_{j}”

# Solve Problem
prob.solve()

# Output Results
print(f”Status: {pulp.LpStatus[prob.status]}”)
print(f”Optimal Cost: {pulp.value(prob.objective)}”)
for i, j in x:
    print(f”Transporting {pulp.value(x[i, j])} units from {i} to {j}”)
```